Forcing of self-excited round jet diffusion flames

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Abstract

In this experimental and numerical study, two types of round jet are examined under acoustic forcing. The first is a non-reacting low density jet (density ratio 0.14). The second is a buoyant jet diffusion flame at a Reynolds number of 1100 (density ratio of unburnt fluids 0.5). Both jets have regions of strong absolute instability at their base and this causes them to exhibit strong self-excited bulging oscillations at well-defined natural frequencies. This study particularly focuses on the heat release of the jet diffusion flame, which oscillates at the same natural frequency as the bulging mode, due to the absolutely unstable shear layer just outside the flame.

The jets are forced at several amplitudes around their natural frequencies. In the non-reacting jet, the frequency of the bulging oscillation locks into the forcing frequency relatively easily. In the jet diffusion flame, however, very large forcing amplitudes are required to make the heat release lock into the forcing frequency. Even at these high forcing amplitudes, the natural mode takes over again from the forced mode in the downstream region of the flow, where the perturbation is beginning to saturate non-linearly and where the heat release is high. This raises the possibility that, in a flame with large regions of absolute instability, the strong natural mode could saturate before the forced mode, weakening the coupling between heat release and incident pressure perturbations, hence weakening the feedback loop that causes combustion instability.

Keywords: non-premixed, laminar, absolute instability, combustion instability
Colloquium: Laminar flames

Total length of paper determined by Method 2 (full 2 column-formatted compilation):
6 pages + 100 mm (single column) = $6 \times 900 + 220 = 5620$ words

1. Introduction

Global self-excited oscillations can be found in both reacting and non-reacting flows. Examples include flickering of a candle flame [1], vortex shedding in bluff body wakes [2] and bulging of a low-density jet [3]. The effect of time-periodic forcing on self-excited non-reacting flows has been studied in some detail, see Ref. [4] for a review. These studies indicate that the self-excited oscillations are relatively insensitive to external forcing (see §1.2). This paper reports an experimental and numerical study into the effect of time-periodic forcing on a self-excited reacting flow, the flickering jet diffusion flame. Our primary aim is to determine how insensitive the self-excited oscillations are to external forcing. Our secondary aim is to examine how the self-excited mode at one frequency affects the heat release response at the forcing frequency. If the fact that a flame is self-excited means that its heat release is less sensitive to external forcing then it could also be less susceptible to combustion instability. Although the diffusion flames studied here are rarely susceptible to combustion instability, the same concept would apply to premixed flames.

1.1. Global modes and local absolute/convective instability

The self-excited flows mentioned above all contain a significant region of absolute instability at an upstream location. The terms 'absolute' and 'convective' describe the instability of the velocity profile at a particular stream-wise location. Perturbations in an absolutely unstable flow grow exponentially upstream and downstream, and eventually contaminate the entire flow. Perturbations in a convectively unstable flow, by contrast, grow downstream only [5] [6]. A flow that is convectively unstable everywhere acts as an amplifier of external perturbations and could therefore be particularly susceptible to combustion instability. On the other hand, a flow that contains a significant region of absolute instability acts as an oscillator. The frequency of this oscillating global mode is determined by the most upstream location of absolute instability, known as the 'wavemaker' region [7]. A global mode's sensitivity to perturbations at the injector lip depends on the position of this wavemaker region. If the flow is absolutely unstable everywhere, the global mode has a large amplitude everywhere and, consequently, is relatively insensitive to external forcing. If, however, there is an appreciable region of convective instability between the injector lip and the wavemaker region, the external forcing can amplify within this region, overwhelm the absolute instability and hence dictate the frequency of the global mode.

In summary, flows that are absolutely unstable everywhere are relatively insensitive to external forcing. Those that are convectively unstable at their base and absolutely unstable downstream are mildly insensitive to external forcing. Those that are convectively unstable everywhere are dominated by external forcing.

1.2. Low density non-reacting jets

Low density non-reacting jets discharging into a quiescent fluid have regions of strong absolute instability at their base due to the axisymmetric Kelvin-Helmholtz (K-H) instability [8]. The K-H instability is caused by the inflexion point in the jet's radial velocity profile and becomes increasingly unstable as the density gradient steepens in the opposite direction to the velocity gradient [9]. This gives rise to a self-excited global mode with a clear spectral peak [10] [11]. Three experimental techniques can confirm that the flow has transitioned to a global mode via a Hopf bifurcation [12] [13] [4]. First, the limit cycle amplitude of this mode should increase in proportion to the square root of the deviation from a control parameter [11]. Second, when the flow is forced periodically, the amplitude at which the frequency of the natural mode, f_n, locks into the forcing frequency, f_f, should increase in proportion to $|f_n - f_f|$ [3]. Third, the transient response should be well described by a Landau equation [14]. This paper investigates an implication of the second technique: that, for forcing amplitudes less than a critical value, the frequency of the global mode is independent of the forcing frequency.

1.3. Jet diffusion flames

In a reacting jet, the flame changes the density profile and hence the velocity profile through the action of buoyancy. If the jet discharges upwards, which is the case studied here, the velocity profile gains two additional inflexion points [15] [16] [17]. Depending on the position of these inflexion points relative to the density profile, the flow can become more unstable or less unstable [9] [18]. This happens because buoyancy alters the velocity profile and therefore changes the K-H instability [19]. This effect does not require a buoyancy term within the perturbation equations of the stability analysis. (Although the buoyancy itself can act on the perturbation equations and alter the instability, this effect is small for the Froude numbers in this study [9]). Crucially, in many jet diffusion flames, such as a methane jet, the inflexion point in the shear layer just outside the flame is absolutely unstable [17]. This gives rise to a strong global mode in this shear layer, which stretches the flame and modulates the heat release in synchronisation with the global mode [18].

It has been known for some time that the flicker of a jet diffusion flame can be suppressed by adding two control cylinders just downstream of the injector [20]. Similarly, vortex shedding behind a cylinder can be suppressed by adding one control cylinder just downstream [21]. Vortex shedding behind a cylinder arises from a region of absolute instability just downstream of the cylinder [12] [2] [22]. The control cylinder changes this region from absolutely to convectively unstable [21]. The fact that the same effect is observed in jet diffusion flames is strong evidence that these flames are also absolutely unstable at their base.

In these jet diffusion flames, the inflexion point that is responsible for absolute instability lies just outside the flame, where buoyancy causes the radial gradients of axial velocity and density to have opposite sign [17]. Conversely, buoyancy causes the inner inflexion point to become more stable (in a non-reacting low density jet, this inflexion point would be absolutely unstable). Consequently, unlike low density non-reacting jets, the frequency of the global mode is relatively insensitive to the velocity and diameter of the inner jet, as is well-documented in the references. Instead, it is very sensitive to the factors affecting buoyancy, such as the gravitational acceleration, g, and the adiabatic flame temperature of the reactants [17]. Ingenious experiments on aircraft and centrifuges have demonstrated this strong dependence on g and weak dependence on other factors [23] [24].

Our aim is to investigate the sensitivity of absolutely unstable jet diffusion flames to external forcing of the jet. We expect these flames to be broadly insensitive to forcing until the forcing amplitude is large enough to cause the natural mode to lock in to the forcing. We will carefully examine the behaviour before lock-in because this will determine how decoupled the heat release can be from the incident pressure perturbations.

2. Methodology, experimental set-up and numerical approach

One non-reacting jet is studied, both numerically and experimentally, in order to calibrate our technique against previous work. Then two similar jet flames are studied, one numerically and one experimentally. Numerical simulations are used when the forcing frequency is very close to the natural frequency, when it is important to have minimal external noise. The experiments are used when noise is less influential in order to cover a wide range of parameters and frequencies.

The experiments are performed on a 6 mm internal diameter contoured nozzle (contraction ratio 52:1), with sinusoidal forcing applied by a loudspeaker mounted upstream of the nozzle exit. This cavity acts as a Helmholtz resonator with natural frequencies of 142 Hz when filled with air, 191 Hz for methane and 380 Hz for helium. These frequencies are sufficiently far from the self-excited frequencies of the helium jet (983 Hz) and methane jet diffusion flame (14 Hz) so as not to affect them. The turbulence intensity is below 0.4% of the mean flow and the velocity profile is flat over the central 4 mm. The non-reacting flow is examined with a hot-wire anemometer (HWA) on the centreline 1.5 diameters downstream of the nozzle exit. The reacting flows are examined with a Phantom v4.2 high speed camera via schlieren (1000Hz) and natural OH* chemiluminescence (600Hz), which is integrated across the flame at axial positions from $2.9x/D$ to $17.8x/D$.

In this paper we do not require absolute measurements of the velocity and heat release themselves but instead are concerned with the relative strengths of peaks in the Power Spectral Densities (PSD) of their fluctuations. For this, we require proxies that increase monotonically with velocity and heat release. For the non-reacting flow, we use the hot wire voltage fluctuation and, for the reacting flow, we use the OH* chemiluminescence fluctuations. Both are sufficiently accurate for comparison of the strengths of the PSD peaks.

The numerical simulations are performed with the low Mach scheme in Ref [9], adapted for a reacting flow. This study aims to capture the effect of forcing, rather than the exact flame shape, so some flame parameters are adjusted for numerical speed. The chemistry is modelled by a simple one step reversible reaction with a Damköhler number $Da = 2 \times 10^5$. A production term $Da\dot{\omega}$ is added to the RHS of the energy equation ($2.2d$ in [9]), where

$$\dot{\omega} = \rho^2 f(Z,T) \exp\left[\frac{-\beta(1-T)}{1-\alpha(1-T)}\right], \quad (1)$$

$$f(Z,T) = \left[Z - \frac{T}{r^*+1}\right]\left[1 - Z - \frac{r^*T}{r^*+1}\right] - \kappa T^2.$$

The heat release parameter, α, is calculated from the adiabatic flame temperatures of the fuel mixtures considered. The Zeldovich number, β, which is typically 8 for real flames, is taken to be 3. This creates a thicker flame, which is easier to resolve numerically. The Reynolds number is 1000 and $Pr = Sc = 0.7$. Since $Le = 1$, two independent variables describe the flame: the mixture fraction, Z, and the reduced temperature, T. The stoichiometric ratio of oxidizer to fuel is r^* and κ, set to 0.01, is an equilibrium constant representing reversible chemistry. For unburned gases with density ratio $S \equiv \rho_{jet}/\rho_{air}$, the density is found from the equation of state:

$$1 = \rho\left[\frac{1-S}{S}Z+1\right]\left[\frac{\alpha}{1-\alpha}T+1\right]. \quad (2)$$

With these parameters, a grid of 1024 (axial) $\times$ 512 (radial) points, on a domain 10 (axial) $\times$ 5 (radial) jet diameters is sufficient to capture 3 to 4 wavelengths of the global instability, while also resolving the flame. A slight co-flow (1% of jet velocity) is added so that the flow is one-way at the bottom boundary. The Froude number, $Fr \equiv u_{jet}/(gd_{jet})^{0.5}$, quantifies the buoyancy.

3. Results

3.1. Non-reacting helium jet

A non-reacting helium jet is studied in order to calibrate against previous work and to examine the behaviour before lock-in more carefully. At $Re = 1100$, the natural mode of the unforced jet is 983 Hz, corresponding to a Strouhal number of 0.27 (mode 1 instability) [11]. When forced, both this natural frequency, f_n, and the forcing frequency of 1023 Hz,
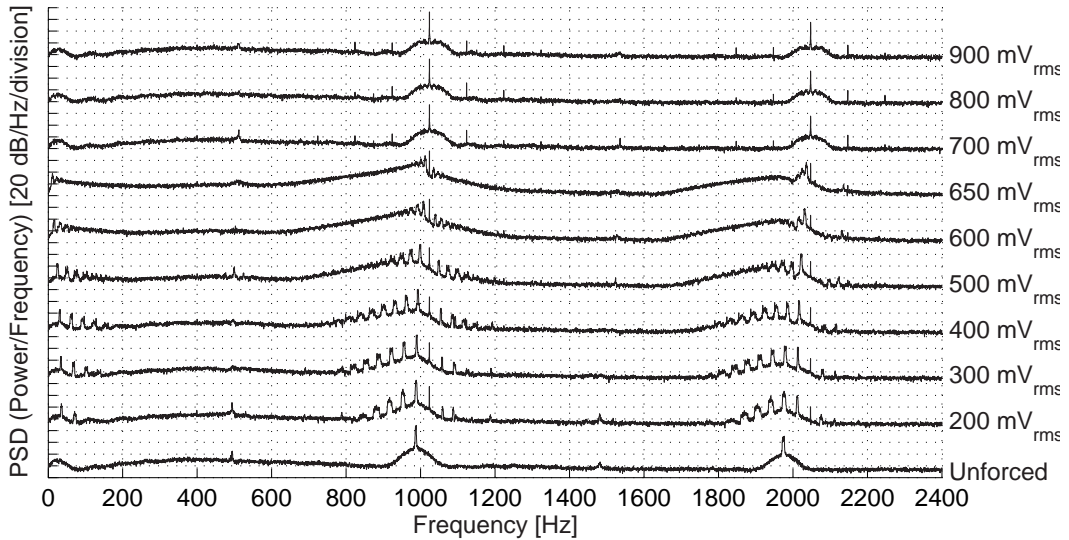
Fig. 1: Power Spectral Density of HWA voltage fluctuations at $(x/D, r/D) = (1.5, 0)$ for a forced helium jet at $Re = 1100$, as a function of the forcing loudspeaker RMS voltage. Here, the natural frequency is 983 Hz and the forcing frequency is 1023 Hz.
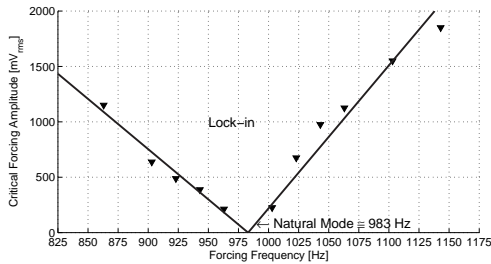

Fig. 2: Critical loudspeaker voltage at which the global mode of the helium jet in Fig. 1 locks into the forcing frequency. The linear dependence on $|f_r - f_f|$ is typical of a Hopf bifurcation.

f_f can be seen in the PSD (Fig. 1). Lock-in is defined as the forcing amplitude at which the natural mode's spectral peak disappears (between 650 and 700 mV$_{rms}$ in Fig. 1). The lock-in amplitude is found to increase in proportion to $|f_n - f_f|$ (Fig. 2), which indicates a Hopf bifurcation to a global mode, as expected. The multiple low frequency peaks in Fig. 1 are caused by non-linear interactions between the forcing frequency and the natural frequency, showing that the flow behaves like a coupled oscillator before lock-in. Figure 2 is similar to that found in [3] but, to the authors' knowledge, this beating has not previously been reported.

The helium jet has also been examined numerically (Fig. 3). Here, the forcing is expressed as a percentage of the jet velocity, rather than as the experimentally-measured loudspeaker voltage. With $S = 0.139$ and $Fr = 19.21$, the non-dimensional natural frequency is $\omega = 1.32$ ($St = 0.21$). The flow is forced 10% above and below this frequency by changing the axial velocity at the nozzle exit to $u_x = (1 + a \sin(\omega t)) f(r)$, where $f(r)$ is Michalke's profile #2 [26]. Four forcing amplitudes are chosen: $a = 0.001, 0.005, 0.01,$ and 0.05. The behaviour of the simulations is very similar to that of the experiments. At low forcing, there is beating between the forcing and natural frequencies and, before lock-in, several peaks appear due to non-linear interactions. By $a = 0.05$, the response has locked into the forcing frequency.

These results confirm that we can detect a global mode in our low density jet and can measure the forcing amplitudes at which this mode locks into the forcing frequency. We also find that, rather than being insensitive to forcing below lock-in amplitudes, there is beating between the natural and forcing frequencies.

3.2. Reacting CH$_4$ jet (experimental)

The methane jet studied here is injected from a contoured nozzle. It is therefore absolutely unstable throughout and has a strong global mode. In this respect it is similar to the non-reacting jet (§3.1). This differs, however, from the methane jet studied by Ref. [17], which was injected with a Poiseuille velocity profile and was convectively unstable for the first jet diameter. We also studied a 30% H$_2$ + 70% N$_2$ jet diffusion flame, whose reaction region lies close to the shear layer. This flame is convectively unstable throughout and therefore does not exhibit a self-excited global oscillation [27] [18]. The results are not presented here because our aim is to focus on the effect of forcing on self-excited flames.
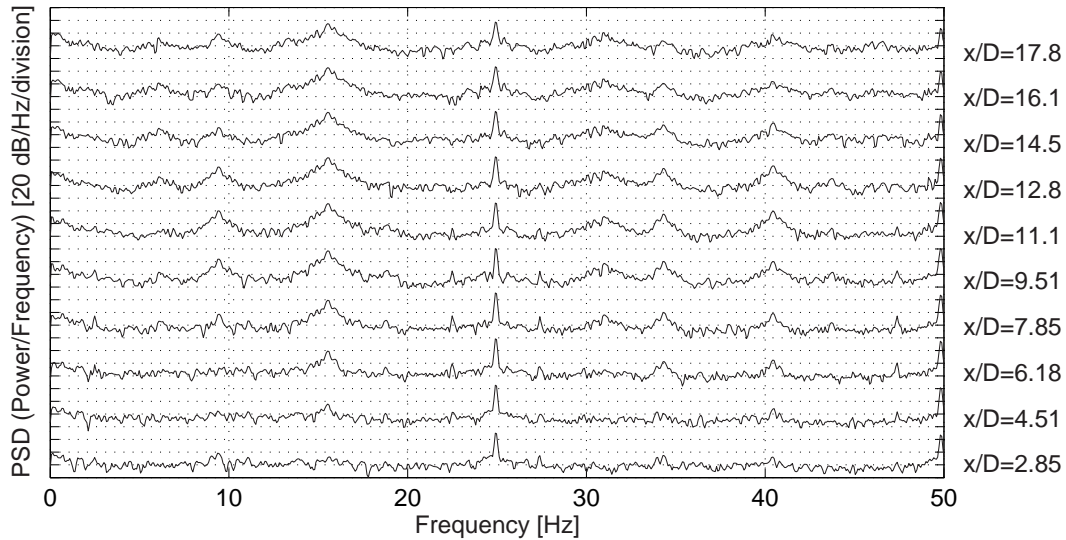
Fig. 7: PSD of OH emission intensity at various x/D when forced at 25.0 Hz and 800 mV ($\sim 80\%$ of the mean jet velocity). The proportion of heat release at the global mode's frequency increases with x/D.

When unforced, the flame has a strong global mode at 14 Hz, as seen in Fig. 4, where the first and fifth frames are at the same phase in the cycle. The inner jet is then forced at 7.0 Hz, 10.0 Hz, 17.0 Hz, 20.0 Hz, 23.0 Hz and 25.0 Hz. The forcing amplitude is increased until the flame lifts off. At low forcing, the outer K-H billows remain at 14 Hz, as seen in the top frames of Fig. 5, where the first and fifth frames are still at the same phase in the natural cycle. At high forcing amplitudes, the outer K-H billows lock into the forcing frequency, as seen in the bottom frames of Fig. 5, where the first and fifth frames are no longer at the same phase in the natural cycle.

The primary research question of this study is: how does the forced mode affect the natural mode? Figure 6 shows the PSD of OH emission at $x/D = 11.1$ for the six forcing frequencies as the forcing amplitude is increased. The natural peak, at 14 Hz, remains robust even at high forcing amplitudes. For forcing below 14 Hz, it never locks into the forcing frequency (consequently, we cannot create a figure similar to Fig. 2). For forcing above 14 Hz, it only locks in at the highest forcing amplitude. Before lock-in, there is beating between the forcing and the natural frequencies, as for the non-reacting jet.

These results raise a second research question: how does the flame's natural mode affect the forced mode? In general, when there is competition between two modes at different frequencies, one will take over and saturate non-linearly as the downstream distance increases [7]. Figure 7 shows the PSD of OH* emission at various x/D when forced at 25 Hz and 800 mV ($\sim 80\%$ of mean jet velocity). At the base of the flame, the natural mode (at 14 Hz) is weak and the forced mode is strong. Farther downstream, however,

the natural mode is strong and the forced mode is noticeably weaker. This downstream region, where the natural mode is beginning to dominate, has the highest OH* emission and is where the perturbation is beginning to saturate non-linearly (Fig. 4). Although not conclusive, the evidence suggests that the natural mode could continue to dominate farther downstream, in the region of high heat release, and thereby weaken the heat release response at the forcing frequency. Further experiments are required to test this thoroughly.

3.3. Reacting H_2/N_2 jet (numerical)

For the simulations, a reacting jet containing 50% H_2+50%N_2 is considered at various values of g (non-dimensionalized with respect to 9.81 ms^{-2}). For this fuel, $S = 0.52$, $\alpha = 0.8546$ and $Z_{st} = 0.304$. As g increases, the inflexion point just outside the flame (§1.3) causes global instability [24]. Table 1 shows that the Strouhal number of the global mode increases as $St \sim g^{0.4}$ (compared with $g^{0.50}$ [24] and $g^{0.67}$ [23]). The wavelength of the global mode decreases correspondingly. This confirms that the global instability arises from the buoyancy-induced inflexion point and that the simulations contain the essential underlying physics. For the simulated forcing experiments we select $Fr = 8$ so that the computational domain contains 2-3 wavelengths.

The central jet is forced in the same way as for the non-reacting jet. Two forcing frequencies are chosen: $\omega = 0.5$ and $\omega = 0.6$, which are quite close to the natural frequency at $\omega = 2\pi St = 0.5428$. A forcing level that causes beating in the non-reacting jet ($a = 0.001$) is found to have no effect on the reacting jet. This concurs with the experimental results,
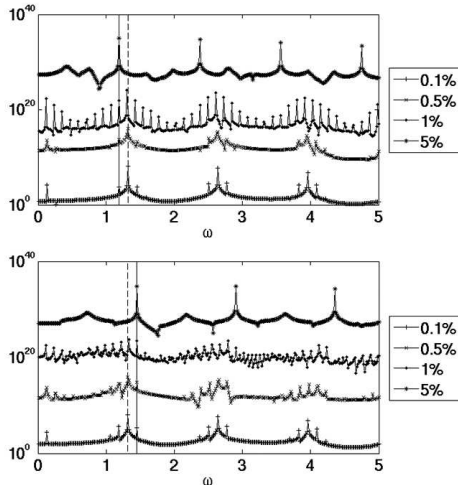
Fig. 3: PSD of mixture fraction at a fixed location $(x, r) = (2.0, 0.5)$ for the simulated helium jet. The flow is forced at low frequency (top) and high frequency (bottom) and at four amplitudes, shown as a percentage of the jet velocity. The global mode's frequency (dotted line) locks into the forcing frequency (solid line) when the forcing reaches 5%. Below that amplitude, the flow beats like a coupled oscillator.

g	Fr	St
1	19.21	stable
2.56	12	0.0633
3.69	10	0.0735
5.77	8	0.0864
10.25	6	0.1102

Table 1: Strouhal number as a function of Froude number for the global mode of a simulated 50% H_2/50% N_2 jet diffusion flame

which also find that the reacting jet is more resistant to forcing than the non-reacting jet.

Four forcing amplitudes are chosen: $a = 0.01, 0.05, 0.1$ and 0.2. The PSD of mixture fraction (Fig. 8) shows peaks at the natural and the forcing frequency. The peak at the forcing frequency takes over as the forcing amplitude increases. The PSD of integrated reaction rate (Fig. 9) shows peaks that can just be distinguished at the higher forcing frequency ($\omega = 0.6$). At the lower forcing frequency ($\omega = 0.5$), the heat release locks into the forcing frequency at these forcing amplitudes. This feature was found at $x/D = 2, 4, 6$ and 8. The simulation did not extend as far downstream as the experimental images so the downstream dominance of the global mode could not be checked.

In summary, the forced simulations show the same qualitative behaviour as the forced experiments. They confirm that the heat release frequency can lock into the forcing frequency when the forcing frequency is close to the frequency of the global mode.
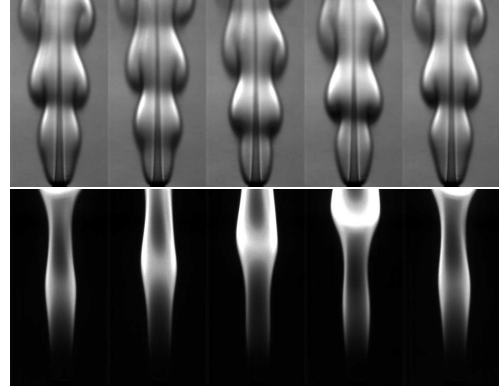

Fig. 4: Schlieren (top) and OH natural emission (bottom) images captured 17.9 ms apart, showing one 14 Hz cycle of the unforced CH_4 jet diffusion flame.

4. Conclusions

We have studied both non-reacting and reacting jets that contain large regions of absolute instability. These regions trigger strong global shear modes at specific natural frequencies. Experiments and numerical simulations show that these jets can lock into an external forcing frequency if the forcing has a sufficiently high amplitude. Before lock-in, the flow's natural mode and the forcing mode behave as a coupled oscillator and there is beating between the two frequencies. For the non-reacting forced jets, both the experiments and the simulations show that the frequency response has several low frequency peaks, due to non-linear interactions between the two frequencies. This, however, is not seen in the reacting jets.

The self-excited oscillations of the reacting jets are much less sensitive to forcing than those of the non-reacting jets. Unless the natural and forcing frequencies are very close, the heat release rate locks into the forcing frequency only at very high forcing amplitudes (of the same order as the jet velocity). Even at these high forcing amplitudes, the natural mode starts to dominate again in the downstream region of the flow, where the perturbation is beginning to saturate non-linearly and where the heat release is high. This raises the possibility that, in a flame with large regions of absolute instability, the strong natural mode could saturate before the forced mode, weakening any coupling between heat release and incident pressure perturbations. In a combustion chamber, this would weaken the feedback loop that causes combustion instability. This has limited applicability to the non-premixed flames studied here, which are rarely susceptible to combustion instability, but could be applied to premixed flames. One of our next steps is to repeat these experiments for a globally-unstable premixed flame.
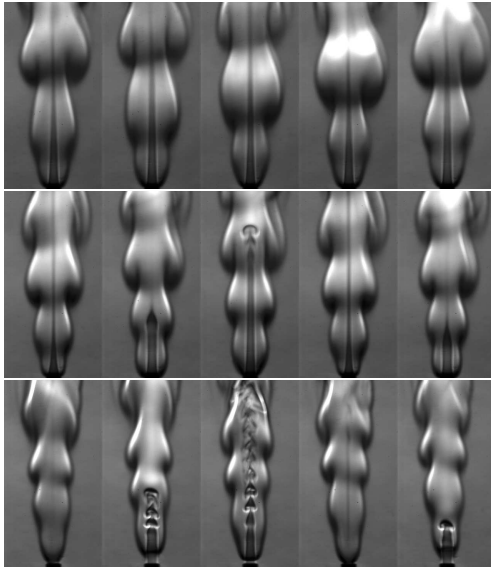
Fig. 5: Schlieren images of the forced CH_4 flame taken 17.9 ms apart. The forcing frequency is 17 Hz and the forcing amplitude is 200 mV (top), 500 mV (middle) and 1000 mV (bottom), corresponding to approximately 20 %, 50 % and 100 % of the mean jet velocity.
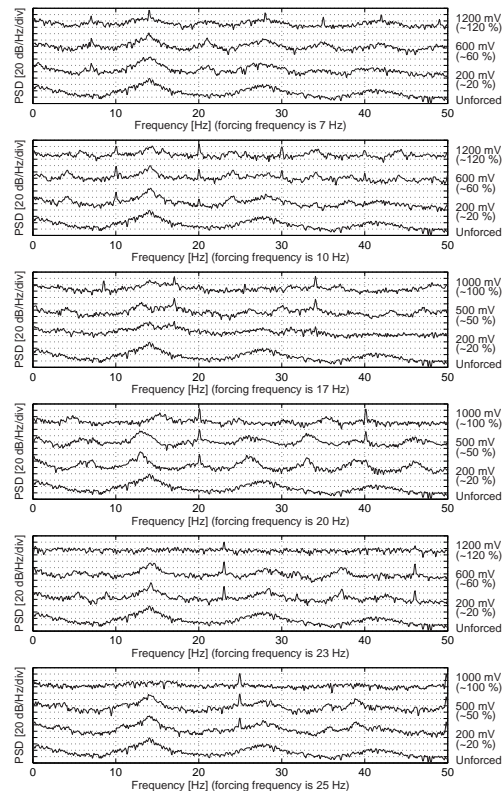
Fig. 6: PSD of OH emission intensity at $x/D = 11.1$ when forced at 7.0 Hz, 10.0 Hz, 17.0 Hz, 20.0 Hz, 23.0 Hz and 25.0 Hz. The global mode, at 14 Hz, is quite robust and is overwhelmed only at the highest forcing amplitude. The figures in brackets are the forcing as a percentage of the mean jet velocity.

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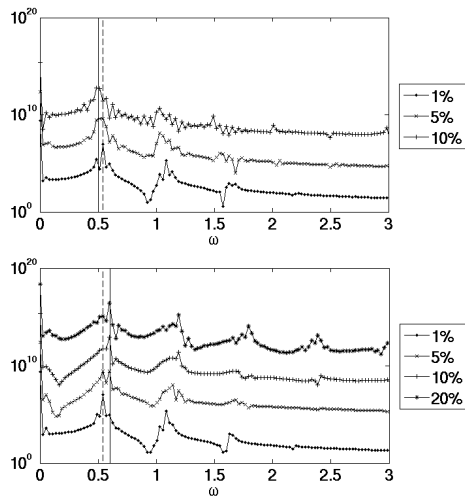
Fig. 8: PSD of the mixture fraction at $(x, r) = (1.992, 0.628)$ for a forced globally unstable numerically-simulated jet diffusion flame as a function of forcing frequency and amplitude. Dotted line: natural frequency. Solid lines: forcing frequency (0.5 in the top plot, 0.6 in the bottom plot). The forcing amplitude, a, is shown as a percentage of the jet velocity.
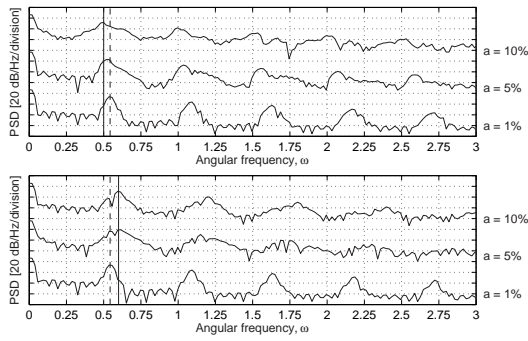

Fig. 9: PSD of the integrated reaction rate $\dot{\omega}$ at $x/D = 4$ for the same flame and forcing as Fig. 8.

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