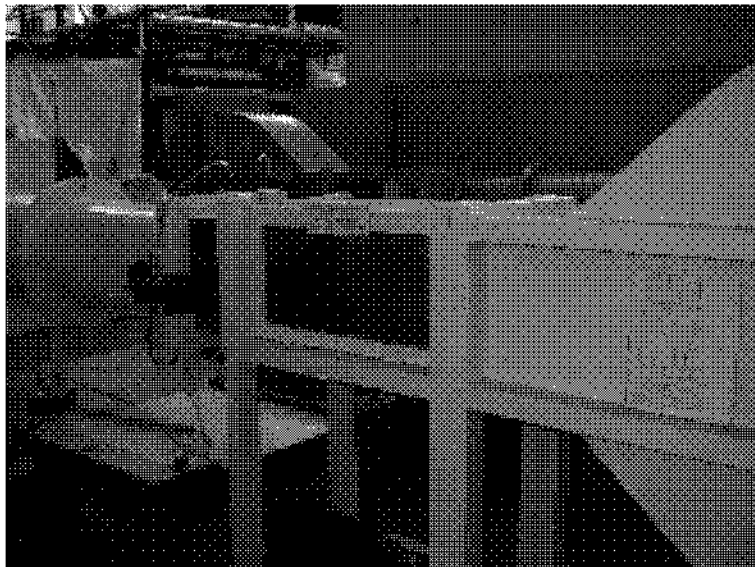


# CHAPTER 7

## THE BOUNDARY LAYER EXPERIMENT

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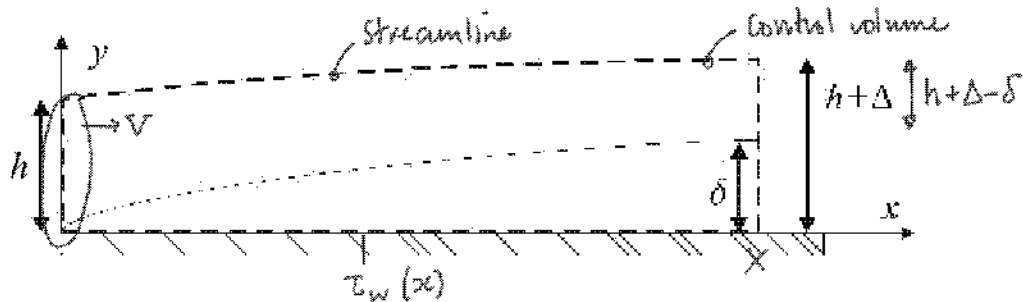
- Recap on laminar boundary layers
- Reynolds number in a boundary layer
- Momentum loss in a boundary layer
- Transition to turbulence in a boundary layer
- Effect of turbulence on a boundary layer
- Comparison of transition and separation
- Boundary layer re-attachment





### 7.3 MOMENTUM LOSS IN A BOUNDARY LAYER

There is a shear stress at the boundary and therefore the fluid loses momentum as the boundary layer grows. The rate of momentum loss can be related to the shear stress. One way to do this calculation is to consider the control volume shown below:



By conservation of mass (per unit length into the page),  $\dot{m}_{out} = \dot{m}_{in}$ :

$$\rho V(h+\Delta-\delta) + \int_0^\delta \rho v(y) dy = \rho V h$$

$\circ$   $\leftarrow$  velocity in the boundary layer

By balance of x-momentum (per unit length into page), the force on the control volume equals the net flux of momentum out:

$$-\int_0^X \tau_w(x) dx = \rho V(h+\Delta-\delta)V + \int_0^\delta \rho v^2 dy - \rho V h V$$

$\circ$  force on c.v.       $\leftarrow$  momentum flux out  $\rightarrow$        $\uparrow$  momentum flux in

Multiplying the mass equation by V and subtracting it from the momentum equation gives:

$$\int_0^X \tau_w dx = V \int_0^\delta \rho v dy - \int_0^\delta \rho v^2 dy$$

$\swarrow$  net force on plate       $\leftarrow$  the momentum that the boundary layer fluid had when it entered the control volume       $\swarrow$  momentum that the boundary layer fluid has when it exits the control volume

The LHS gives the total force that the plate (from  $x = 0$  to  $x = X$ ) exerts on the fluid. The terms on the RHS concern the fluid within the boundary layer. The last term on the RHS is the momentum that this fluid has when it leaves the control volume. The first term on the RHS is the momentum that this fluid *had* before it entered the control volume. The difference is therefore the net flux of momentum out. This works for both laminar and turbulent boundary layers.

With this relationship we can calculate the growth rate of a boundary layer. For example, a simple model of the velocity profile of a laminar boundary layer is:

$$v_x = v(y) = V \left[ \frac{3y}{2\delta} - \frac{1}{2} \left( \frac{y}{\delta} \right)^3 \right]$$

In this model, the wall shear stress is:

$$\tau_w = \mu \left. \frac{\partial v_x}{\partial y} \right|_{\text{wall}, y=0} = \frac{3\mu V}{2\delta}$$

The momentum that the boundary layer fluid had before entering the control volume is:

$$V \int_0^\delta \rho v dy = \frac{5}{8} \rho V^2 \delta$$

The momentum that the boundary layer fluid has on leaving the control volume is:

$$\int_0^\delta \rho v^2 dy = \frac{17}{35} \rho V^2 \delta$$

Putting all this together gives:

$$\begin{aligned} \frac{3\mu V}{2} \int_0^x \frac{dx}{\delta} &= \frac{5}{8} \rho V^2 \delta - \frac{17}{35} \rho V^2 \delta = \frac{39}{280} \rho V^2 \delta \\ \Rightarrow \int_0^x \frac{dx}{\delta} &= \frac{13}{140} \frac{\rho V}{\mu} \delta \end{aligned}$$

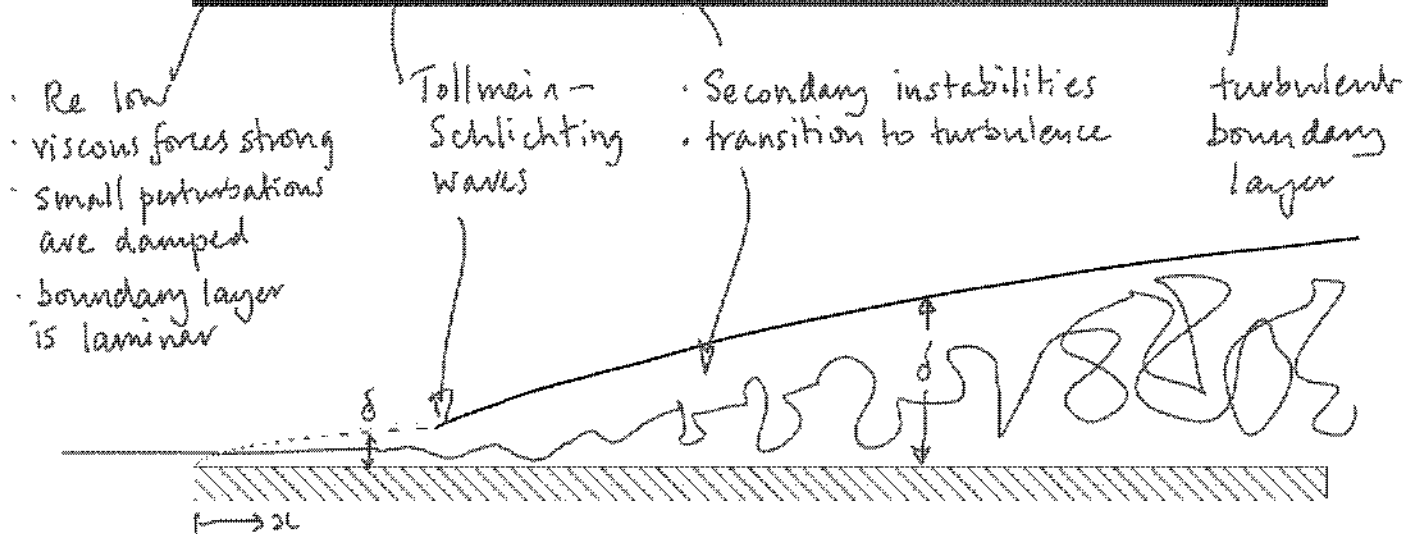
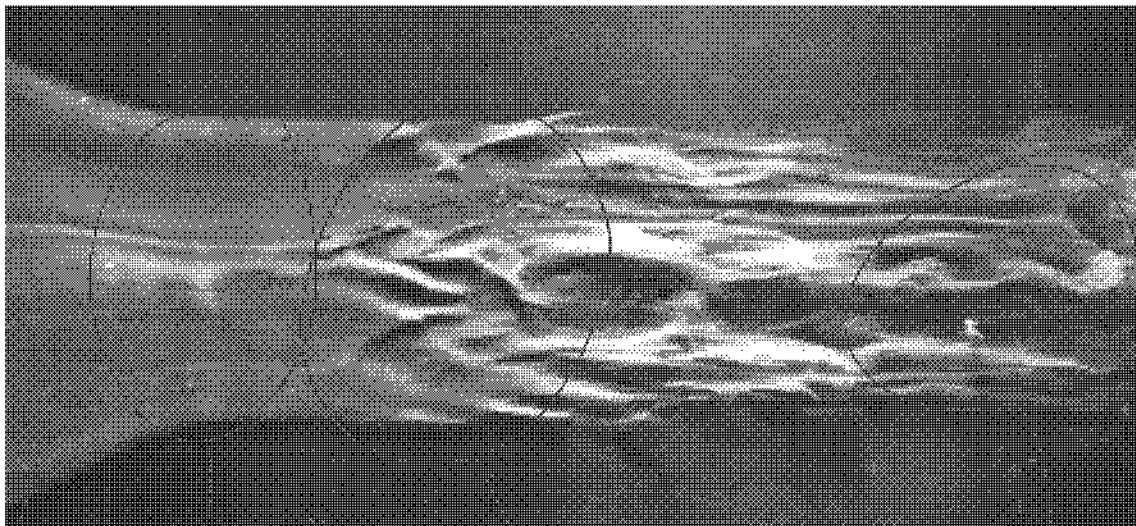
Differentiating with respect to  $x$  gives:

$$\begin{aligned} \frac{1}{\delta} &= \frac{13}{140} \frac{\rho V}{\mu} \frac{d\delta}{dx} \\ \Rightarrow \delta \frac{d\delta}{dx} &= \frac{d}{dx} \left( \frac{\delta^2}{2} \right) = \frac{140}{13} \frac{\mu}{\rho V} \\ \Rightarrow \delta^2 &= \frac{280}{13} \frac{\mu}{\rho V} x \\ \Rightarrow \frac{\delta}{x} &= \frac{4.64}{\sqrt{Re_x}} \leftarrow \text{for this particular velocity profile} \right. \\ &\quad \left. \leftarrow \frac{\rho V x}{\mu} \right. \end{aligned}$$

So, by combining the equation for the momentum loss in a boundary layer with an equation for the boundary layer profile, we can calculate the boundary layer growth in the  $x$ -direction. Note that this derivation is very sensitive to the velocity gradient at the wall.

## 7.4 TRANSITION TO TURBULENCE IN A BOUNDARY LAYER

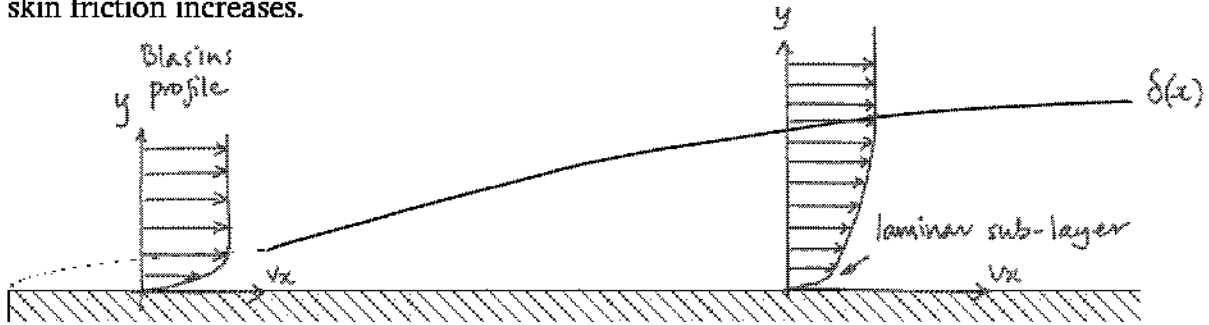
At low Reynolds numbers, the viscous forces are so strong that they damp down any perturbations in the fluid motion and therefore the boundary layer remains laminar. At increased Reynolds numbers, the inertial effects (mass  $\times$  acceleration) become more dominant and cause the boundary layer flow to become unstable. Above a certain Reynolds number, boundary layer disturbances called Tollmein–Schlichting waves grow. In flows that start nearly perfectly laminar, turbulence is initiated by these waves. In flows that already contain small velocity fluctuations, turbulence is initiated by transient growth of these small fluctuations, bypassing the formation of Tollmein–Schlichting waves.



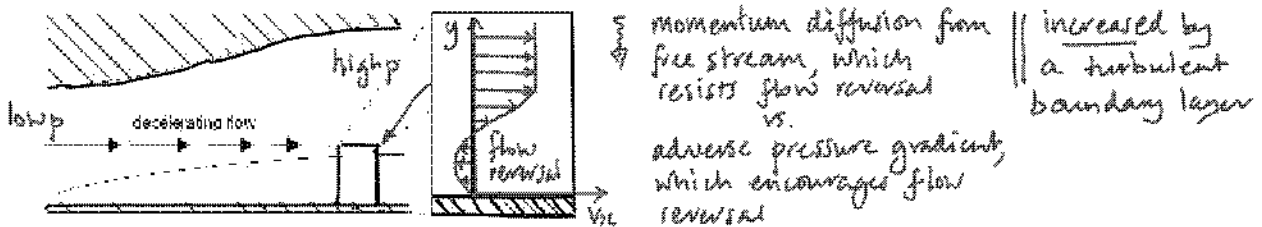
You listen to the turbulent boundary layer with a stethoscope in the boundary layer experiment. Although the motion in the bulk of the boundary layer is turbulent, there is always a very thin region next to the body where the viscous forces are so strong that they damp down all the turbulence. This is called the laminar sub-layer.

## 7.5 EFFECT OF TURBULENCE ON A BOUNDARY LAYER

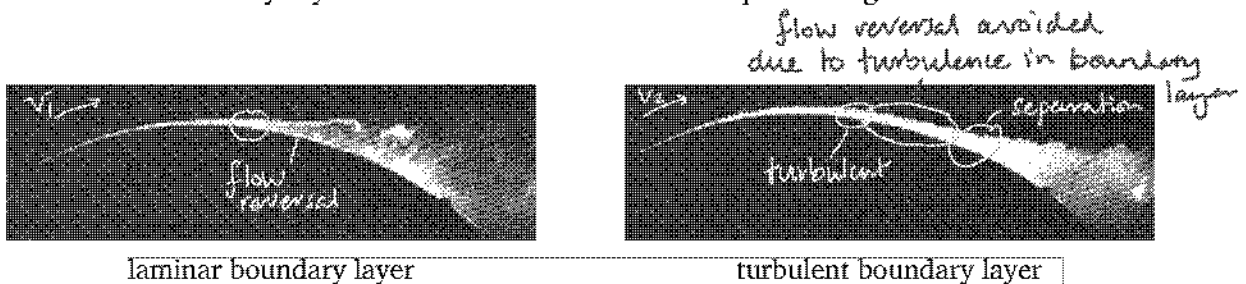
Turbulence increases the rate of momentum transfer between the surface and the free stream, in a manner similar to an increase in the viscosity. This has three direct effects: the boundary layer grows more quickly, the velocity profile is fatter, and the skin friction increases.



The increase in momentum transfer has another very important consequence. In chapter 4 we saw that the flow inside a boundary layer can reverse direction in the presence of a strong adverse pressure gradient and that this causes *separation* of the boundary layer from the body.



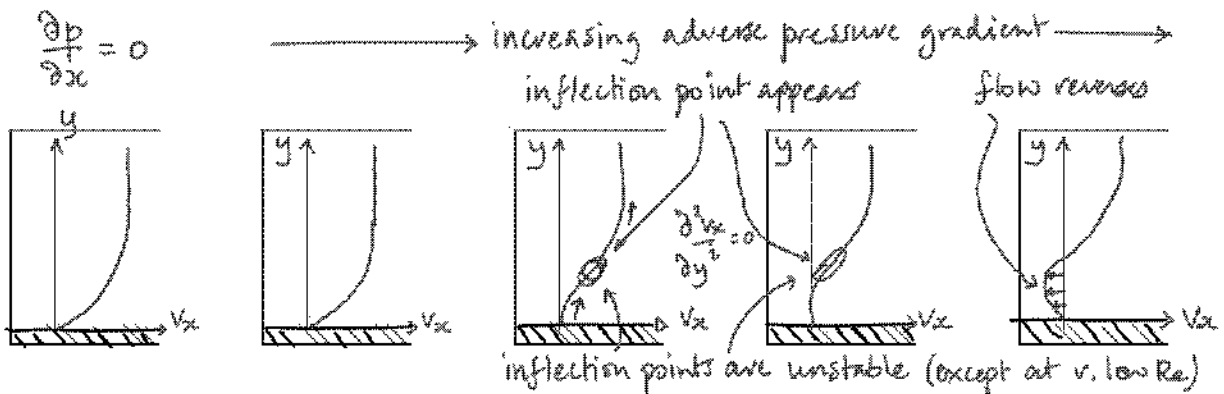
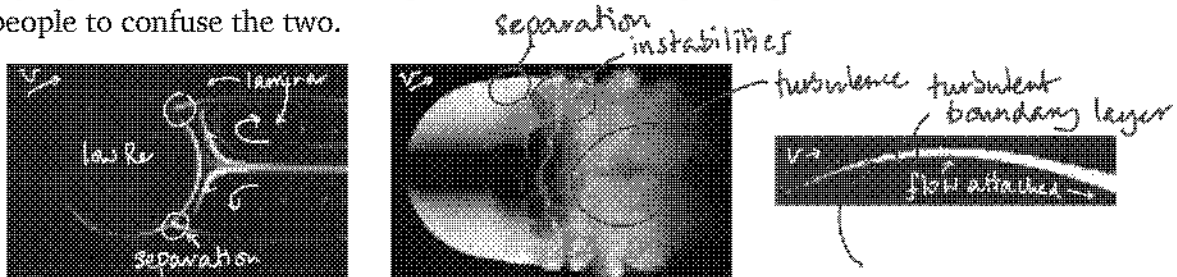
There is a competition between momentum transfer from the free stream (which resists flow reversal) and the adverse pressure gradient (which encourages flow reversal). Turbulence *increases* momentum transfer from the free stream and therefore it makes a boundary layer *more resistant* to an adverse pressure gradient.



Therefore a turbulent boundary layer can *reduce* form drag. This is why golf balls are dimpled. We examine this further in chapter 8.

## 7.6 COMPARISON OF SEPARATION AND TRANSITION TO TURBULENCE

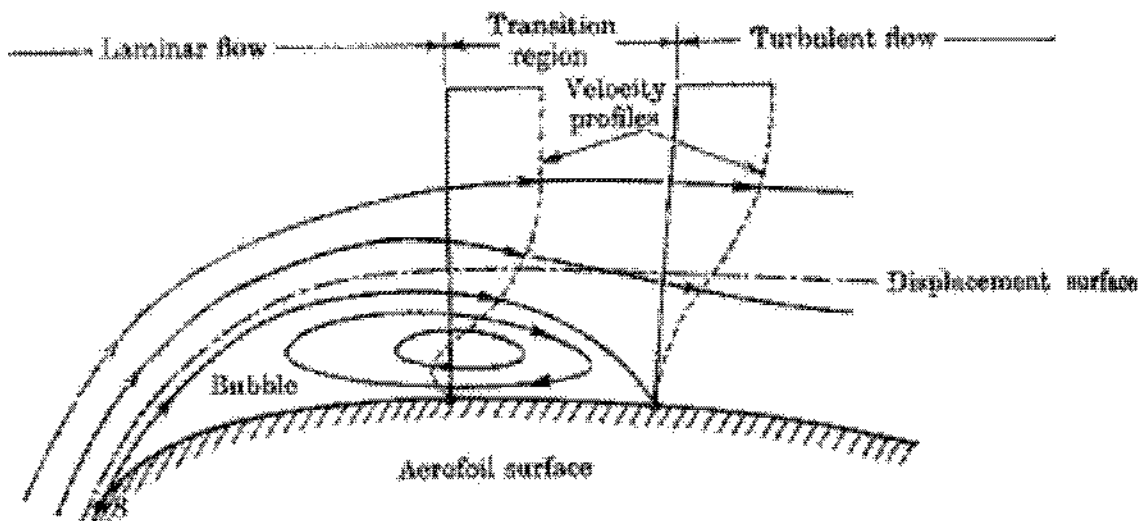
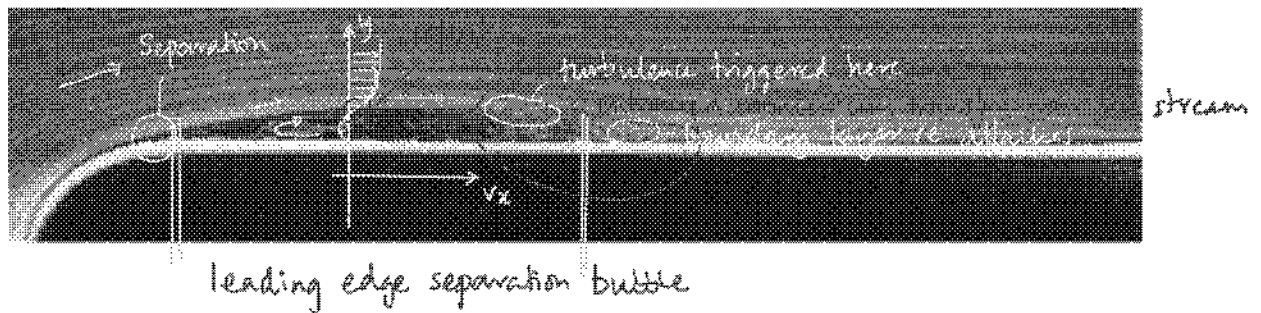
Boundary layer separation and boundary layer transition to turbulence are entirely different phenomena. Nevertheless, transition to turbulence often occurs in boundary layers that are about to separate or that have just separated. This leads some people to confuse the two.



In the majority of flows, separation causes almost instantaneous transition to turbulence - picture (b) above. In very viscous separated flow, however, the viscosity is high enough to damp down any perturbations and the flow remains laminar - picture (a) above. If there is little or no adverse pressure gradient the boundary layer will become turbulent without any danger of separation - picture (c) above.

## 7.7 BOUNDARY LAYER RE-ATTACHMENT

Sometimes a boundary layer separates, becomes turbulent and then re-attaches due to the increased momentum transfer. This leads to a small separation bubble



It is very dangerous in flight because the wing suddenly loses all lift if the bubble bursts.