

# CHAPTER 6

## ONE DIMENSIONAL PIPE FLOW

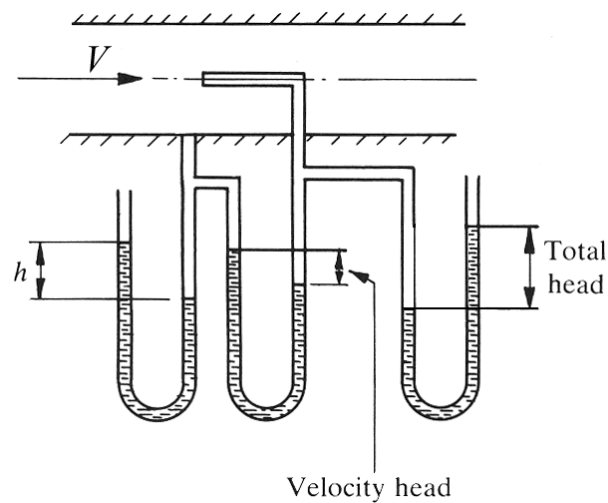
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- Static pressure, stagnation pressure, and total pressure
  - Total pressure loss across an orifice plate
  - Total pressure loss along a pipe
  - Total pressure changes across pipe components
  - Total pressure and mechanical work
  - Network analysis
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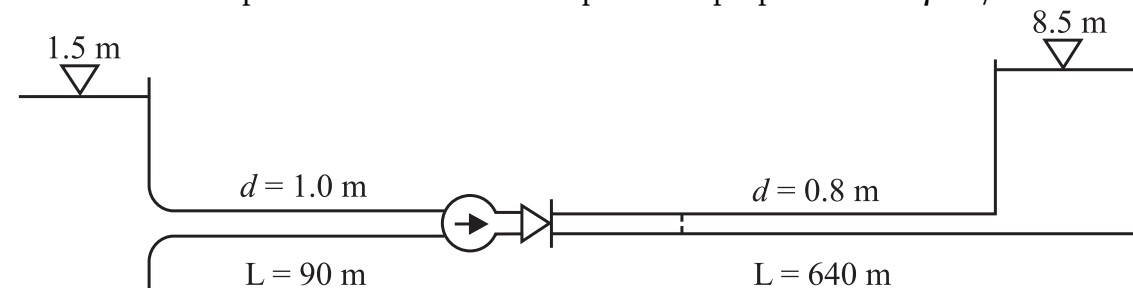
*Internal Flow Systems* by D. S. Miller is an excellent source of practical information on internal flow (ISBN 0-947711-77-5 and classmark TA 379/ TA 293 in the CUED library)

## 6.1 STATIC PRESSURE, STAGNATION PRESSURE, AND TOTAL PRESSURE

At a point in a moving fluid the *static* pressure,  $p$ , is the pressure measured by a probe that does not change the speed of the flow, e.g. at point A on the diagram. The *stagnation* pressure,  $p_0$ , is that measured by a probe that lets the flow come to rest without loss of mechanical energy, e.g. at point B on the pitot tube (there will be more detail on this in chapter 10). The two pressures are related by Bernoulli's equation.



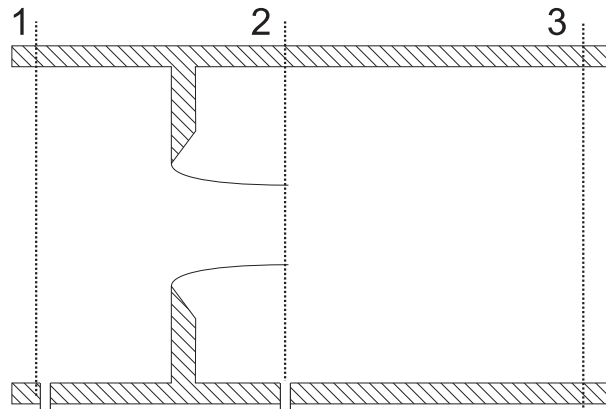
The *stagnation* pressure is the pressure measured at a stagnation point. The *total* pressure,  $p + \rho V^2/2 + \rho gh$ , includes the height,  $h$ , measured relative to an arbitrary datum level and is *not* the pressure measured at a stagnation point. In a network of pipes we find it useful to follow the total pressure, rather like following the voltage in an electrical network. When divided by  $\rho g$ , this is also known as the *head*. The total pressure is conserved if there is no loss of mechanical energy in the flow. It drops as the flow loses mechanical energy through pipes, bends, orifice plates and other components. It rises as the flow goes through pumps. At sufficiently high Reynolds number the total pressure loss in each component is proportional to  $\rho V^2/2$ .



When the height,  $h$ , does not change, the change in total pressure equals the change in stagnation pressure. In sections 6.2 to 6.9, we will assume that the height does not change through the components being considered.

## 6.2 TOTAL PRESSURE LOSS ACROSS A HORIZONTAL ORIFICE PLATE

In the pipe flow experiment an orifice plate is used to measure the flowrate. The flowrate is calculated from the static pressure drop across the plate. The pressure drop depends on the size of orifice, the sharpness of the edges and where the pressure tappings are placed relative to the plate. Orifice plates are calibrated experimentally but here we use a simple model to estimate the pressure drop. We assume that the velocity is, on average, uniform and steady across section 1, section 3 and the central jet in section 2.



The area of the central jet adjusts until the *static* pressure is uniform across the whole of section 2. If we assume that there are no viscous losses between section 1 and section 2 then Bernoulli can be applied along a streamline.

$$p_1 + \frac{1}{2}\rho V_1^2 = p_2 + \frac{1}{2}\rho V_2^2$$

$$\Rightarrow p_1 - p_2 = \frac{1}{2}\rho V_1^2 \left( \frac{V_2^2}{V_1^2} - 1 \right)$$

(The orifice is horizontal so we can ignore height changes.) If we knew  $A_2/A_1$  we could calculate  $V_2/V_1$  from conservation of mass between sections 1 and 2:

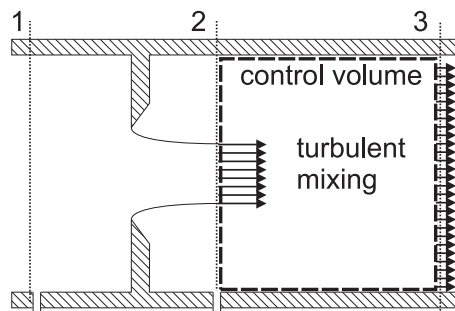
$$=$$

$$\Rightarrow p_1 - p_2 = \frac{1}{2}\rho V_1^2 \left( \left( \frac{A_1}{A_2} \right)^2 - 1 \right)$$

In most situations we will not know  $A_2/A_1$ . However, the velocity ratio  $V_2/V_1$  remains a function only of the orifice diameter and shape so the term in brackets is a constant that can be determined experimentally. Thus  $V_1^2$  and the flowrate can be found by measuring the static pressure drop  $p_1 - p_2$ . This is the main function of orifice plates.



Between section 2 and section 3 the jet mixes turbulently. Turbulent eddies decay to smaller and smaller eddies, which quickly lose their mechanical energy through viscous dissipation, so Bernoulli cannot be applied. However, the steady flow momentum equation and conservation of mass can be used between these two sections\*<sup>1</sup>.



From the steady flow momentum equation, the net momentum flux equals the pressure difference multiplied by the area on each side of the control volume:

$$p_2 - p_3 = \frac{\dot{m}}{A_3}(V_3 - V_2) = \rho V_3(V_3 - V_2) = \rho V_3^2 \left(1 - \frac{V_2}{V_3}\right)$$

By conservation of mass,  $V_3$  is equal to  $V_1$ , so the total static pressure drop is:

$$p_1 - p_3 = \frac{1}{2}\rho V_1^2 \left(1 - \frac{V_2}{V_1}\right)^2 = \frac{1}{2}\rho V_1^2 K$$

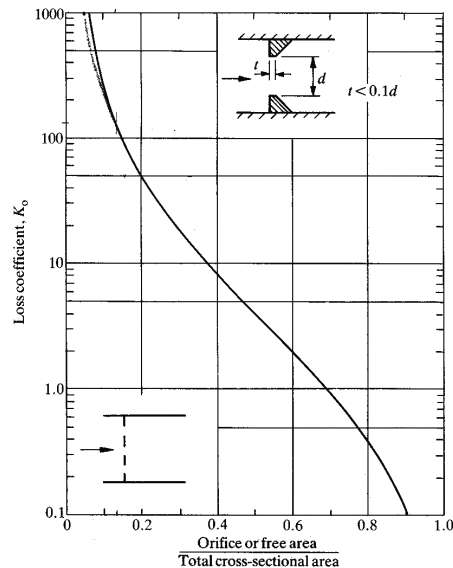
The stagnation pressure drop is the same because  $V_3$  equals  $V_1$ :

$$p_{01} - p_{03} = p_1 + \frac{1}{2}\rho V_1^2 - p_3 - \frac{1}{2}\rho V_3^2 = p_1 - p_3 = \frac{1}{2}\rho V_1^2 K$$

The *loss coefficient*,  $K$ , is a function of the orifice diameter and shape.  $K$  is a constant for flows at high Reynolds number.

<sup>1</sup>\*See Derivation 1 at the back of the handout for all the intermediate steps in this calculation

If we repeat the analysis, accounting for height changes between 1 and 3, we must replace  $(p_{01} - p_{03})$  in the above expression with  $(p_{tot,1} - p_{tot,3})$ . The loss coefficient,  $K$ , is tabulated or plotted in books for different orifice shapes:



### 6.3 TOTAL PRESSURE LOSS ALONG A PIPE

In chapter 5 we derived an expression for the pressure gradient along a pipe in terms of the friction coefficient,  $c_f$ :

$$\frac{dP}{dx} = -\frac{\rho V^2}{R} c_f$$

If this pressure gradient is uniform then:

If the pipe has uniform cross-sectional area and the flow inside is fully-developed then the average velocity,  $V$ , is uniform. Consequently, the stagnation pressure drop in a horizontal pipe is exactly equal to the static pressure drop:

$$p_{02} - p_{01} = p_2 + \frac{1}{2}\rho V_2^2 - p_1 - \frac{1}{2}\rho V_1^2 = p_2 - p_1 =$$

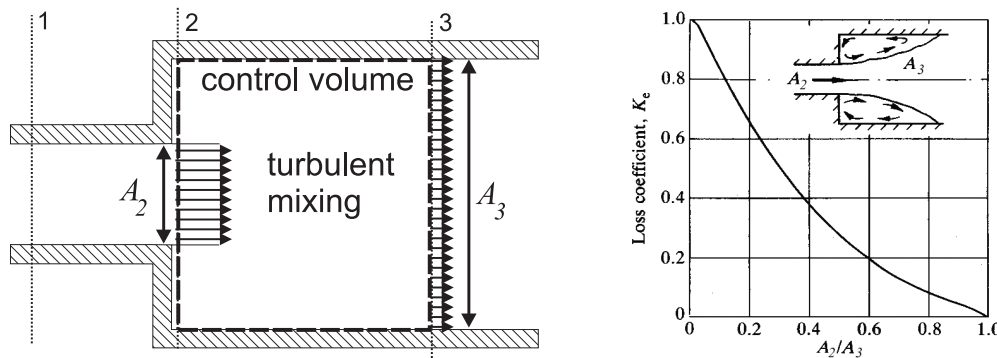
In a non-horizontal pipe, we must replace  $p_{02} - p_{01}$  with  $p_{tot,2} - p_{tot,1}$ . There are actually two definitions of the friction coefficient. The other is denoted  $f$  and is equal to  $4c_f$ . In this course we call  $f$  the *friction factor* although in some books it too is called the friction coefficient.

## 6.4 TOTAL PRESSURE LOSS AT A SUDDEN EXPANSION

There is a total pressure loss when the cross-sectional area of a pipe suddenly increases. If we analyse<sup>2</sup> this in the same way as a horizontal orifice plate we calculate that the stagnation pressure drop is given by:

$$p_{02} - p_{03} = \frac{1}{2} \rho V_2^2 \left( 1 - \frac{A_2}{A_3} \right)^2 = \frac{1}{2} \rho V_2^2 K$$

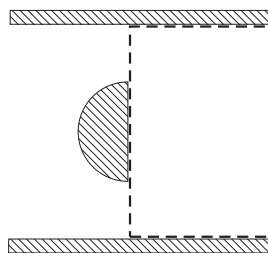
where  $K$  is a new loss coefficient. We can compare this value of  $K$  with the experimental values that are shown on the right. When height changes are included, we must replace  $(p_{02} - p_{03})$  with  $(p_{tot,2} - p_{tot,3})$ .



If a pipe discharges into a reservoir without a gradual expansion of the cross-sectional area, then  $A_3$  tends to infinity and the loss coefficient,  $K$  is equal to 1.

## 6.5 THE PRESSURE AGAINST A WALL

Students are often surprised that the pressure against a wall is assumed to be that of the fluid next to the wall. Some think that the pressure there should be zero.

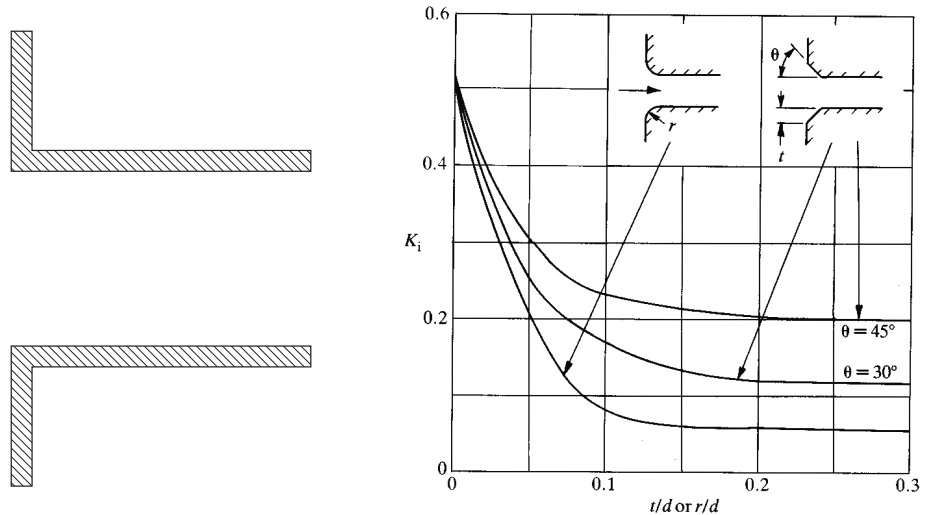


If the wall and the fluid are at thermal equilibrium, molecules hit the wall, stick to it for a while, and are released back into the fluid with the same energy with which they hit the wall. Statistically, this is equivalent to replacing the wall with a fluid at the same temperature and pressure as the fluid next to the wall. You can think of the wall as a ‘pressure mirror’. If, instead, we were to assume that the pressure is zero at the wall, it would be equivalent to replacing the wall with a vacuum. In other words, all molecules would disappear on hitting the wall. This is evidently not a good model of a wall’s real behaviour.

<sup>2</sup>\*See Derivation 2 in the back of this handout for the intermediate steps in this calculation

## 6.6 TOTAL PRESSURE LOSS AT A PIPE ENTRANCE

There is a similar total pressure loss at the entrance to a pipe:



## 6.7 TOTAL PRESSURE CHANGES ACROSS GENERAL PIPE COMPONENTS

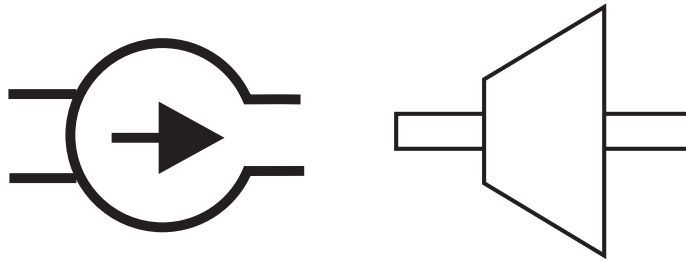
In summary, all pipe components, such as valves, junctions and bends cause a total pressure loss. At high Reynolds number the flow is turbulent and this loss is equal to  $K\rho V^2/2$  where  $K$  is the loss coefficient.  $K$  has been measured experimentally for all components and is tabulated in book such as *Internal Flow Systems* by D. S. Miller:

**Table 12.4.** Loss coefficients for composite diffusers with thick inlet boundary layers (values in parentheses are for thin inlet boundary layers)

No.	$K_{td}$	Arrangement	No.	$K_{td}$	Arrangement
1	0.74 (0.72)		11	0.75 (0.64)	
2	0.72 0.93		12	0.75	
3	0.64 (0.57)		13	0.52	

## 6.8 PUMPS AND TURBINES

Pumps do mechanical work on a fluid and cause a total pressure rise. Turbines, on the other hand, extract mechanical work from a fluid and cause a total pressure loss. The exact mechanisms of this are described in the third year (3A3 Compressible Flow) and the fourth year (Turbomachinery). There are always some losses in such systems due to irreversibility.



## 6.9 TOTAL PRESSURE AND MECHANICAL WORK

A drop in total pressure in the fluid corresponds to a loss of mechanical energy by the fluid. The mechanical energy may have been converted to internal energy through an irreversible thermodynamic process such as viscous dissipation (see chapter 10). Alternatively, it may have exerted shaft power,  $\dot{W}_x$ , on its surroundings via a device such as a turbine.

For an irreversible process, the total pressure change across a control volume is related to the shaft power transferred from the fluid by:

$$\frac{\dot{m}}{\rho}(p_{tot_{out}} - p_{tot_{in}}) = -\dot{W}_x$$

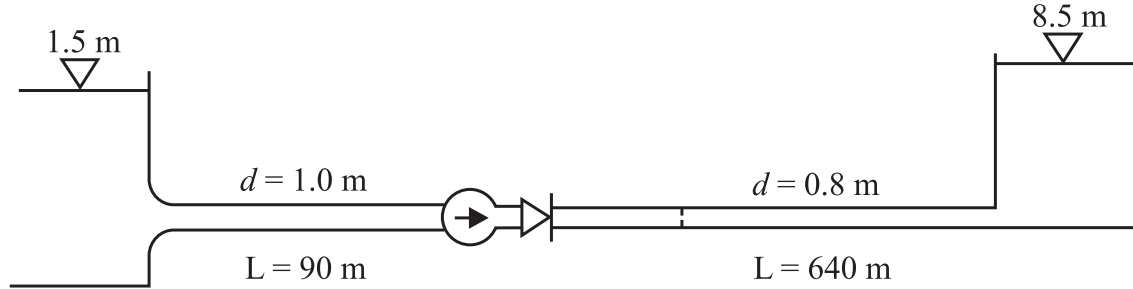
In chapter 10 we will relate this equation, which deals with mechanical energy, to the Steady Flow Energy Equation, which deals with both mechanical and thermal energies. For now will note that if there are any irreversibilities within the control volume, mechanical energy is lost to thermal energy and the equation becomes:

Similarly, if mechanical shaft power is exerted on the fluid then the total pressure rises. Again, the power exerted is equal to the volumetric flowrate multiplied by the total pressure change.



## 6.10 NETWORK ANALYSIS

In a network of pipes, we follow the total pressure through the components of the network. This leads to an equation, or set of equations, for the unknowns in the network. In the example below, we work out the power of the pump that is required to lift water from the left tank, whose surface is 1.5 m above a datum level, to the right tank, whose surface is at 8.5 m. Both surfaces are at atmospheric pressure,  $p_a$ .



Component	diameter (m)	density (kgm <sup>-3</sup> )	velocity (ms <sup>-1</sup> )	Re	K	$\Delta p_{tot}$ (Nm <sup>-2</sup> )	$\Delta$ head (m)
pipe inlet	1.0	1000	3.50	$3.2 \times 10^6$	0.1	613	0.06
pipe	1.0	1000	3.50	$3.2 \times 10^6$	1.35	8270	0.84
pump	-	-	-	-	-	$x$	$x/\rho g$
valve	0.8	1000	5.47	$4.0 \times 10^6$	0.5	7480	0.76
pipe	0.8	1000	5.47	$4.0 \times 10^6$	12	179500	18.30
orifice	0.8	1000	5.47	$4.0 \times 10^6$	0.5	7480	0.76
pipe outlet	0.8	1000	5.47	$4.0 \times 10^6$	1.0	14960	1.53

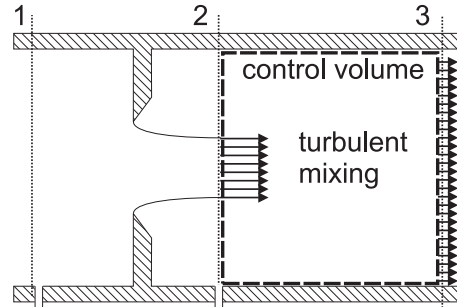
(for pipes,  $K = 4C_f L/D$  with  $c_f = 3.75 \times 10^{-3}$ )

Position	Total pressure (calculation) (Nm <sup>-2</sup> )	Total pressure (Nm <sup>-2</sup> )
In left tank ( $V = 0$ )	$p_a + \rho gh = p_a + 1000 \times 9.81 \times 1.5$	$p_a + 14715$
At entry to pump	$p_a + 14715 - 613 - 8270$	$p_a + 5832$
At exit of pump		$p_a + 5832 + x$
In right tank ( $V = 0$ )	$p_a + 5832 + x - 7480 - 179500 - 7480 - 14960$	$p_a + x - 203588$
Required in right tank	$p_a + \rho gh = p_a + 1000 \times 9.81 \times 8.5$	$p_a + 83385$

Equating the pressures in the right tank gives the total pressure rise in the pump:

The volumetric flowrate is  $2.75 \text{ m}^3\text{s}^{-1}$  so the required power from the pump is:

## DERIVATION 1 - HORIZONTAL ORIFICE PLATE



From the steady flow momentum equation, the net momentum flux equals the pressure difference:

$$A_3 p_2 - A_3 p_3 = \dot{m} V_3 - \dot{m} V_2$$

$$\Rightarrow p_2 - p_3 = \frac{\dot{m}}{A_3} (V_3 - V_2) = \frac{\rho A_3 V_3}{A_3} (V_3 - V_2) = \rho V_3^2 \left(1 - \frac{V_2}{V_3}\right)$$

$V_3$  is equal to  $V_1$  because of conservation of mass so the total static pressure drop is:

$$\begin{aligned} p_1 - p_3 &= (p_1 - p_2) + (p_2 - p_3) \\ &= \frac{1}{2} \rho V_1^2 \left( \left[ \frac{V_2}{V_1} \right]^2 - 1 \right) + \rho V_1^2 \left( 1 - \frac{V_2}{V_1} \right) \\ &= \frac{1}{2} \rho V_1^2 \left( 1 - 2 \frac{V_2}{V_1} + \left[ \frac{V_2}{V_1} \right]^2 \right) \\ &= \frac{1}{2} \rho V_1^2 \left( 1 - \frac{V_2}{V_1} \right)^2 \\ \Rightarrow p_1 - p_3 &= \frac{1}{2} \rho V_1^2 K \end{aligned}$$

where the *loss coefficient*,  $K$ , is  $(1 - V_2/V_1)^2$ .

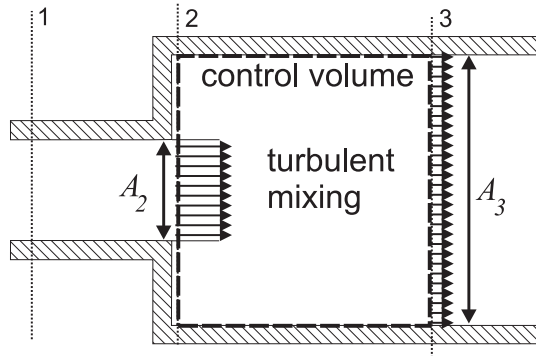
The total stagnation pressure drop is:

$$\begin{aligned} p_{01} - p_{03} &= p_1 + \frac{1}{2} \rho V_1^2 - p_3 - \frac{1}{2} \rho V_3^2 \\ &= p_1 - p_3 \\ &= \frac{1}{2} \rho V_1^2 K \end{aligned}$$

because  $V_1$  is equal to  $V_3$ .

## DERIVATION 2 - HORIZONTAL ABRUPT EXPANSION

There is a stagnation pressure loss when the cross-sectional area of a pipe suddenly increases.



Conservation of mass:

$$V_2 A_2 = V_3 A_3 \quad (6.1)$$

Steady flow momentum equation:

$$p_2 A_3 + \dot{m} V_2 = p_3 A_3 + \dot{m} V_3 \quad (6.2)$$

Re-arranging equation (6.2) gives:

$$p_2 - p_3 = \frac{\dot{m}}{A_3} (V_3 - V_2) = \frac{\rho A_2 V_2}{A_3} (V_3 - V_2) = \rho V_2^2 \left( \frac{A_2 V_3}{A_3 V_2} - \frac{A_2}{A_3} \right)$$

Substituting equation (6.1) into this expression gives:

$$p_2 - p_3 = \rho V_2^2 \left( \left[ \frac{A_2}{A_3} \right]^2 - \frac{A_2}{A_3} \right) \quad (6.3)$$

The stagnation pressure drop is given by:

$$\begin{aligned} p_{02} - p_{03} &= p_2 + \frac{1}{2} \rho V_2^2 - p_3 - \frac{1}{2} \rho V_3^2 \\ &= p_2 - p_3 + \frac{1}{2} \rho (V_2^2 - V_3^2) \\ &= p_2 - p_3 + \frac{1}{2} \rho V_2^2 \left( 1 - \left[ \frac{V_3}{V_2} \right]^2 \right) \end{aligned}$$

Substituting equation (6.1) and equation (6.3) into this expression gives:

$$\begin{aligned} p_{02} - p_{03} &= \rho V_2^2 \left( \left[ \frac{A_2}{A_3} \right]^2 - \frac{A_2}{A_3} \right) + \frac{1}{2} \rho V_2^2 \left( 1 - \left[ \frac{A_2}{A_3} \right]^2 \right) \\ &= \frac{1}{2} \rho V_2^2 \left( 1 - 2 \frac{A_2}{A_3} + \left[ \frac{A_2}{A_3} \right]^2 \right) \\ &= \frac{1}{2} \rho V_2^2 \left( 1 - \frac{A_2}{A_3} \right)^2 \end{aligned}$$