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Non-normality in combustion–acoustic interaction in diffusion flames: a critical revision

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Perturbations in a non-normal system can grow transiently even if the system is linearly stable. If this transient growth is sufficiently large, it can trigger self-sustained oscillations from small initial disturbances. This has important practical consequences for combustion–acoustic oscillations, which are a persistent problem in rocket and aircraft engines. Balasubramanian & Sujith (*J. Fluid Mech.*, vol. 594, 2008, pp. 29–57) modelled an infinite-rate chemistry diffusion flame in an acoustic duct and found that the transient growth in this system can amplify the initial energy by a factor, G_{max}, of the order of 10^5 to 10^7. However, recent investigations by L. Magri and M. P. Juniper have brought to light certain errors in that paper. When the errors are corrected, G_{max} is found to be of the order of 1 to 10, revealing that non-normality is not as influential as it was thought to be.

Key words: Acoustics, Combustion, Flames

1. Results and discussion

Recent investigations have brought to light certain errors in Balasubramanian & Sujith (2008, labelled B&S in this note). We use the same model, discretization and non-dimensionalization as in B&S. The required corrections to B&S are listed below.

(*a*) The analytical steady solution, Z_{st} (appendix B, p. 54), obtained by separation of variables, is

$$Z_{st} = X_i(1-\alpha) - Y_i\alpha - \frac{2}{\pi}(X_i+Y_i)\sum_{n=1}^{+\infty}\frac{\sin(n\pi\alpha)}{n(1+b_n)}\cos(n\pi y_c)(\mathrm{e}^{a_{n1}x_c} + b_n\mathrm{e}^{a_{n2}x_c}), \quad (1.1)$$

where

$$a_{n1} \equiv \frac{Pe}{2} - \sqrt{\frac{Pe^2}{4} + n^2\pi^2}, \quad a_{n2} \equiv \frac{Pe}{2} + \sqrt{\frac{Pe^2}{4} + n^2\pi^2}, \quad (1.2)$$

$$b_n \equiv -\frac{a_{n1}}{a_{n2}} \exp\left(-2L_c\sqrt{\frac{Pe^2}{4} + n^2\pi^2}\right). \quad (1.3)$$

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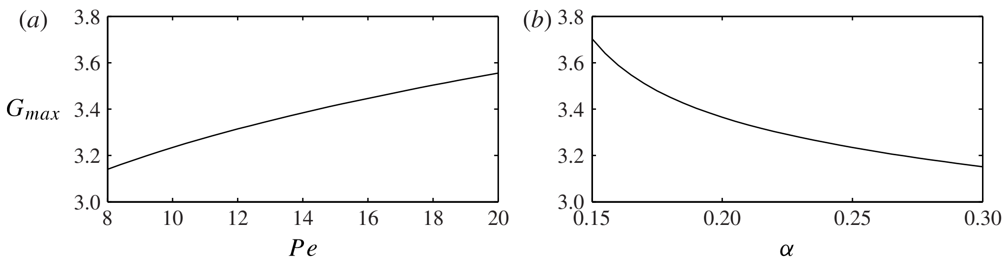

FIGURE 1. The growth factor, G_{max}, as a function of (*a*) the Péclet number, Pe, and (*b*) the fuel slot half-width, α. Panel (*a*) has $\alpha = 0.25$, and (*b*) has $Pe = 10$.

The non-dimensional coordinates of the combustion domain are x_c and y_c.

(*b*) The expressions for $C_m^{(n)}$ and W_{mk} (equation (7), p. 36) are $2/L_c$ times the original terms, due to the Galerkin projection. Furthermore, $W_{mk} = 1/L_c$ when $k = m$.

(*c*) The variable Y_1 on the right-hand side of the terms R_{nm} and J_{nm} (equation (13), p. 37) is Y_i.

(*d*) The variable $\dot{Q}_{av}$ (equations (18) and (19), p. 39) is to be divided by 2 due to non-dimensionalization over the cross-sectional area.

(*e*) The multiplying factor in front of matrix $\boldsymbol{M}_1$ (appendix B, p. 54) is $1/((T_i + T_{ad})/2)$.

(*f*) The expression for matrix $\boldsymbol{B}_{NN}$ (appendix B, p. 54) is $\boldsymbol{B}_{NN} = -\boldsymbol{D} + \boldsymbol{A}_1 - \boldsymbol{A}_2 + \boldsymbol{A}_3 - \boldsymbol{A}_4 + \boldsymbol{A}_5$, where

$$\boldsymbol{A}_5 = \frac{1}{(T_i + T_{ad})/2}[0\ 0\ 0\ \ldots\ 0\ \sin(\pi x_f)\ \sin(2\pi x_f)\ \ldots\ \sin(K\pi x_f)]^{\mathrm{T}} \times [J_{00}\ \ldots\ J_{0M}\ 0\ \ldots\ 0] \tag{1.4}$$

and K is the number of Galerkin modes for acoustic discretization.

(*g*) The damping terms in the matrix $\boldsymbol{S}$ (appendix B, p. 55) are $+2\pi\xi_1, +4\pi\xi_2, \ldots, +2K\pi\xi_K$.

(*h*) The numerator of matrix $\boldsymbol{A}_4$ (appendix B, p. 55) is 1 due to non-dimensionalization over the cross-sectional area of the duct.

We perform computations with 50×50 Galerkin modes in the flame domain and six modes in the acoustic domain. When we increase the number of Galerkin modes to 70×70 in the flame and 12 in the acoustics, the eigenvalues and singular values change by less than 15 %. The fixed parameters are: the fuel mass ratio, $Y_i = 3.2$; the oxidizer mass ratio, $X_i = 3.2/7$; and the average temperature, $T_{av} = 1/0.685$. We set the damping coefficients to $c_1 = 0.013$ and $c_2 = 0.08$ in order to have marginally stable systems. The nonlinear behaviour of this thermo-acoustic system is not considered because it has been fully characterized by Illingworth, Waugh & Juniper (2013).

Figure 1 shows the growth factor, G_{max}, as a function of (*a*) the Péclet number, Pe, and (*b*) the non-dimensional half-width of the fuel slot, α. (Note that we use the same norm as B&S, even though Chu's norm would be a more appropriate measure of the energy (Chu 1965).) In both cases, $1 < G_{max} \lesssim 10$. These plots can be compared with figures 9 and 10 in B&S. Figure 2 shows the eigenvalues and the pseudospectra for this thermo-acoustic system. These can be compared with figure 11 in B&S. The pseudospectra around the most unstable eigenvalues are nearly concentric circles whose values decrease rapidly as the distance from the eigenvalue increases. This is a further demonstration that the system is only weakly non-normal,

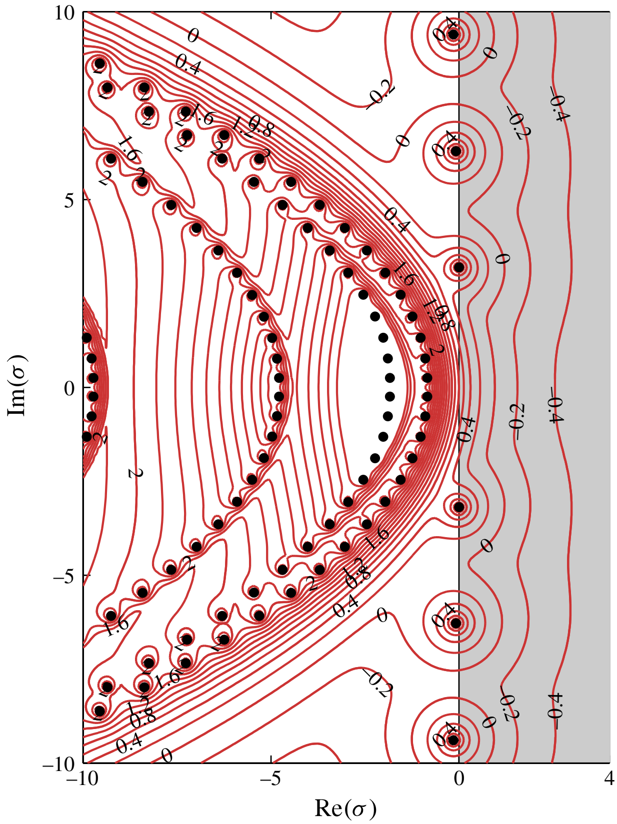

FIGURE 2. (Colour online) Logarithm of the pseudospectra, $\log_{10}(\epsilon)$. The parameters are the same as figure 1, with $\alpha = 0.25$ and $Pe = 10$. The dominant eigenvalue is $\sigma = -0.003 \pm 3.193\mathrm{i}$.

because a marginally stable but highly non-normal system would have pseudospectra that protrude significantly into the unstable half-plane (Trefethen & Embree 2005). Nevertheless, it is worth noting that Juniper (2011) showed that even a small amount of non-normality can make a system somewhat more susceptible to triggering.

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