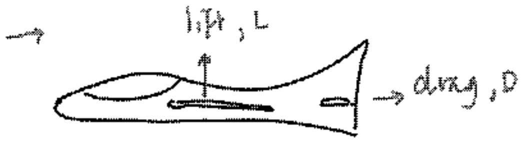
Chapter 8

External flow and drag

- Lift and drag
- Flows at very low Reynolds number (creeping flow)
- Flows at low Reynolds number
- Flows at high Reynolds number
- Mechanisms of drag reduction
- Vortex shedding
- Inviscid flow and Hele Shaw cells

8.1 LIFT AND DRAG

In many situations we do not need to know all the forces acting on an object and we do not need to know exactly what is going on in the flow. We need to know only the net force of the fluid on the object. On an aeroplane, for example, we need to know the net upwards force (the lift) and the net backwards force (the drag).



$$C_L \equiv \frac{L}{\frac{1}{2}\rho V^2 A}$$

$$C_D \equiv \frac{D}{\frac{1}{2}\rho V^2 A}$$

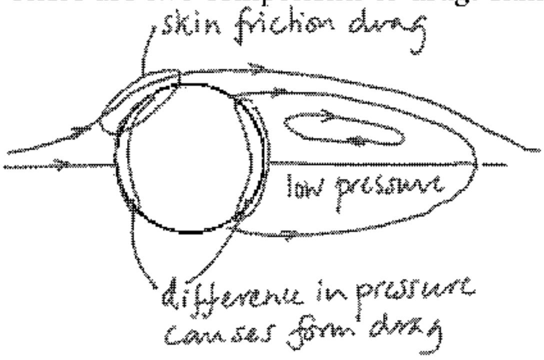
As for pipe flow, we can express the lift and drag in units of $\rho V^2/2$. These are the lift and the drag coefficients. If the flow is incompressible (i.e. at low Mach number) they are functions of the shape of the object, its roughness, the angle of attack and the Reynolds number. If the flow is compressible we need to include the Mach number as well.

C_L, C_D depend on Mach, Reynolds, angle of attack, surface roughness

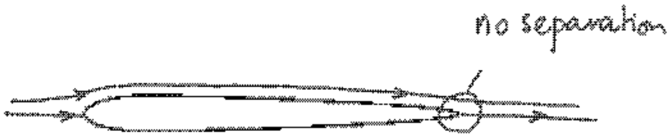
momentum loss through boundary layer

caused by the low pressure region behind a bluff body

There are two components to drag: skin friction and form drag:



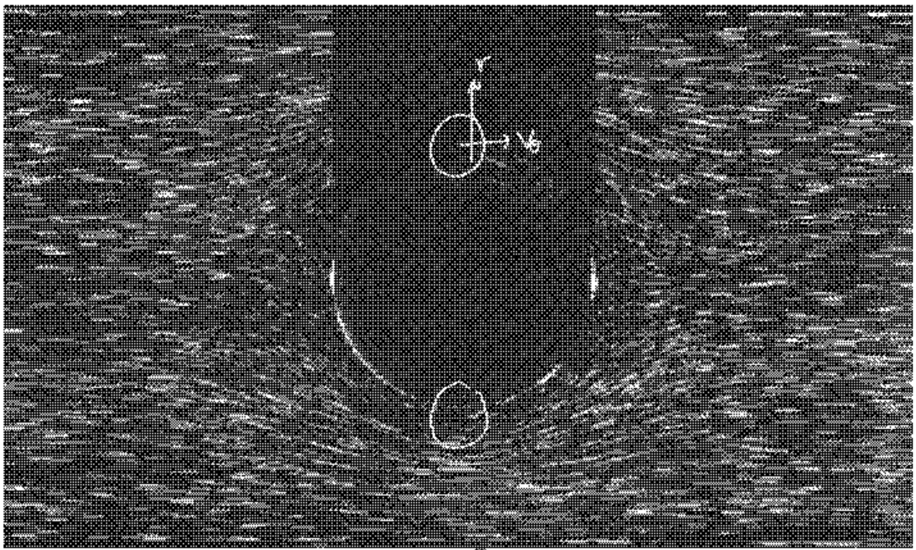
- low skin friction
- high form drag

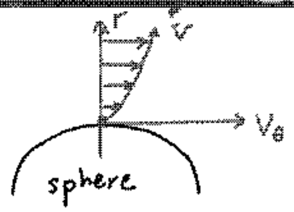


- all drag is caused by skin friction

8.2 Flow at very low Reynolds number - creeping flow

In the spheres experiment you drop a number of different spheres into columns of oil and measure the time of descent. A sphere is spherically symmetric so there is no force perpendicular to the flow (lift) but there is a backwards force, the drag.





At very low Reynolds number (Re 1) the flow is very slow, so the inertial forces are much smaller than the viscous forces. The inertial terms are negligible in the Navier-Stokes equation (which is $\mathbf{f} = m\mathbf{a}$ for a viscous fluid):

$$\mathbf{a} = \frac{1}{m}\mathbf{f}$$

$$\frac{D\mathbf{v}}{Dt} = \frac{\partial \mathbf{v}}{\partial t} + \mathbf{v}\cdot\nabla\mathbf{v} = -\frac{1}{\rho}\nabla p + \frac{\mu}{\rho}\nabla^2\mathbf{v}$$

(pressure; viscous)

and it reduces to:

$$\nabla p \simeq \mu \nabla^2 \mathbf{v}$$

For the flow around a sphere, this has an analytical solution, in the same way that the laminar flow between two flat plates has an analytical solution. It is known as Stokes Flow. For a sphere of radius R in a flow of velocity V:

$$
\begin{aligned}
v_r &= V\cos\theta\left(1-\frac{3R}{2r}+\frac{R^3}{2r^3}\right) \qquad \text{radial} \\
v_\theta &= -V\sin\theta\left(1-\frac{3R}{4r}-\frac{R^3}{4r^3}\right) \qquad \text{azimuthal}
\end{aligned}
$$

The viscous drag[*1] on the sphere is $6\pi\mu RV$. The drag coefficient is therefore:

$$
C_d = \frac{\text{drag}}{\frac{1}{2}\rho V^2 \pi R^2} = \frac{6\pi\mu RV}{\frac{1}{2}\rho V^2 \pi R^2} = \frac{24}{Re} \qquad Re \equiv \frac{\rho V D}{\mu}
$$

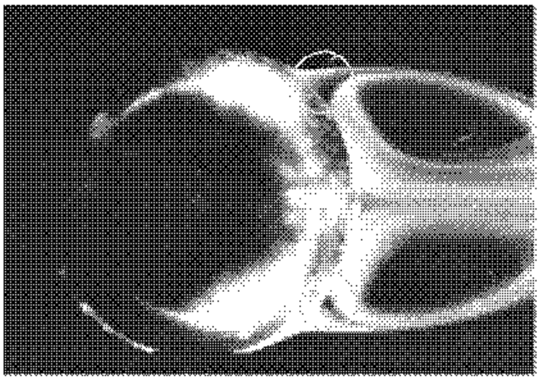
In Stokes flow, all the drag is due to skin friction. There is no form drag because the flow does not separate. The velocity increases with distance away from the surface. The pressure gradient pushes the flow round in the direction of flow. Therefore there are no adverse pressure gradients and no separation of the boundary layer. It is very different from inviscid (i.e. potential) flow, even though the streamlines, at first sight, look similar.

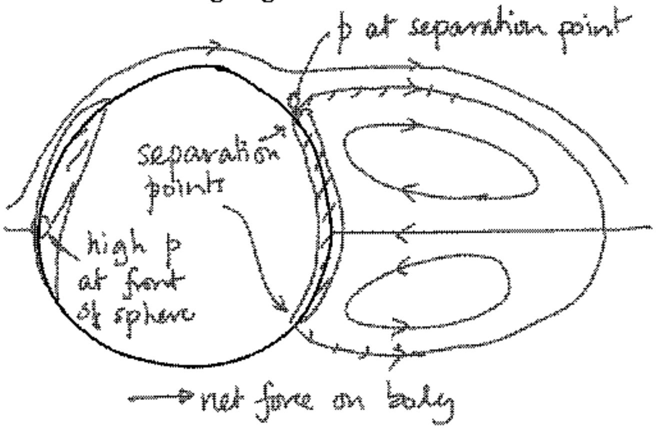
Flows at very low Reynolds number can seem very strange to us because we rarely see them day-to-day.

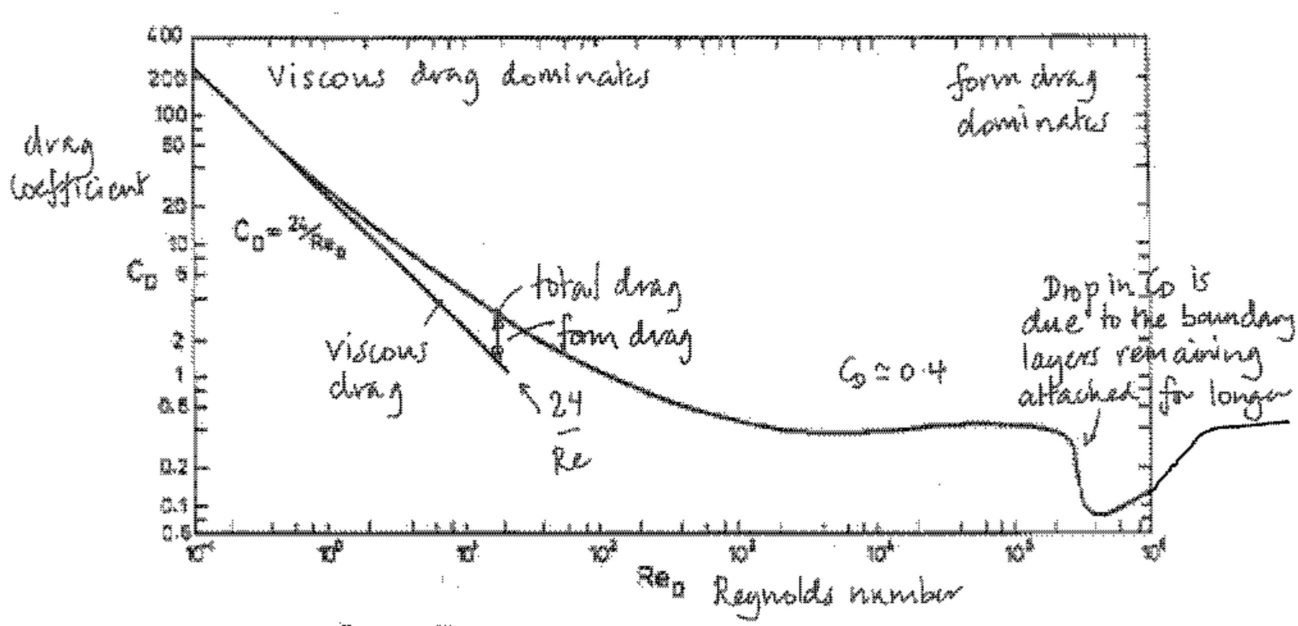
[1*] see Faber, *Fluid Dynamics for Physicists* p230 for a clear derivation of this

8.3 Flow at fairly low Reynolds number

In the flow around a sphere at a Reynolds number of 100, the boundary layer separates just behind the shoulder and a toroidal recirculating region forms.





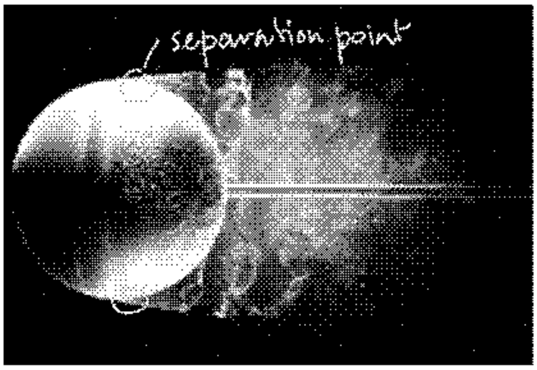


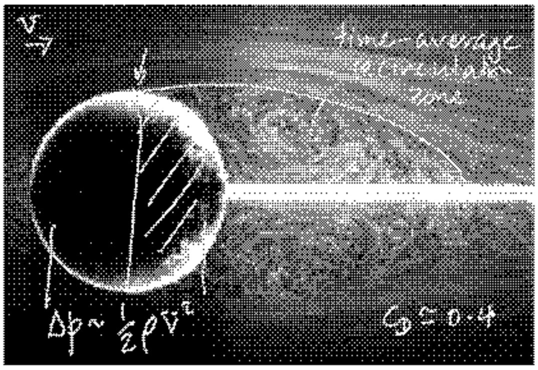
Drag coefficient of spheres as a function of Reynolds number.

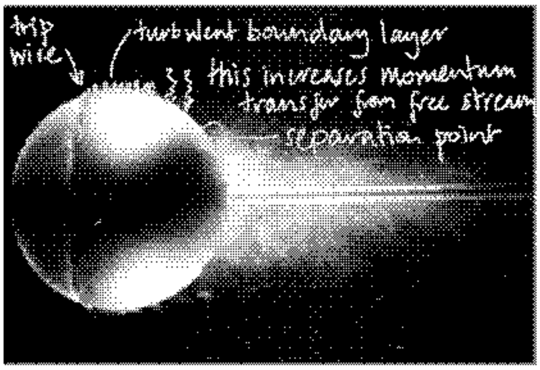
The point of separation is determined by the usual competition between (a) diffusion of momentum from the free stream due to viscosity, which delays separation and (b) the adverse pressure gradient at the back of the sphere, which encourages separation. As the Reynolds number increases, the viscous forces decrease relative to the inertial forces (and therefore relative to the pressure gradient), so the point of separation moves upstream, towards the equator.

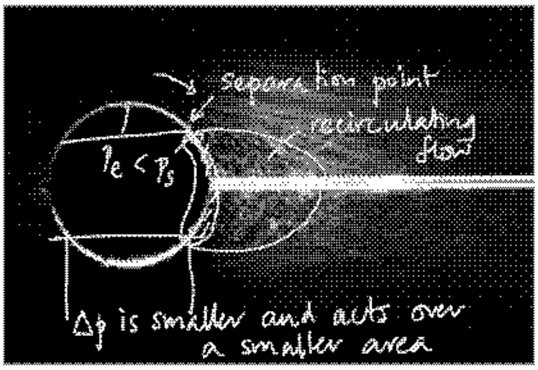
8.4 Flows at higher Reynolds number

As the Reynolds number is increased, the form drag increases relative to the skin friction. Beyond a Reynolds number of 1000, the skin friction is negligible. Between $Re = 1000$ and $Re = 200000$, the point of separation remains very near the equator so the drag coefficient remains approximately constant at $C_D = 0.4$.









In the above photos, the boundary layer is tripped to turbulence with a wire because the wind tunnel could not reach very high Reynolds numbers.

8.6 Drag reduction - streamlining

On the one hand, form drag increases in proportion to the cross-sectional area of the separated region behind a body and in proportion to ρV^2. On the other hand, skin friction increases in proportion to the surface area of the body and in proportion to μV:

form drag ~ (wake area D^2) * dynamic pressure $\frac{1}{2}\rho V^2$

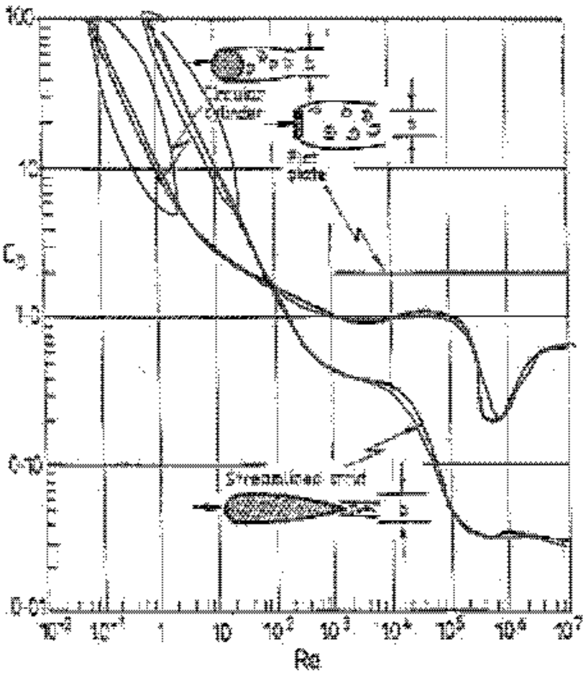
$$C_D \sim \frac{D^2 * \frac{1}{2}\rho V^2}{\frac{1}{2}\rho V^2 D^2} \sim 1$$

skin friction ~ (skin surface area D^2) * wall shear stress

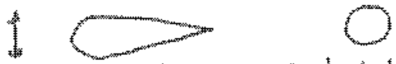
$$\sim D^2 * \mu \frac{\partial v}{\partial r} \sim D^2 \mu \frac{V}{D} \sim D\mu V$$

$$C_D \sim \frac{D\mu V}{\frac{1}{2}\rho V^2 D^2} \sim \frac{\mu}{\rho V D} \sim \frac{1}{Re}$$

Many important applications (e.g. cars and aeroplanes) operate at high Reynolds number where form drag is much greater than skin friction. Therefore the first priority is to reduce form drag. This is achieved by delaying separation of the boundary layers, which requires adverse pressure gradients to be as gentle as possible:

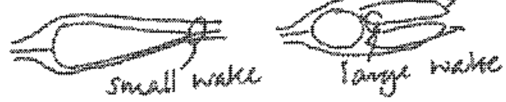


low Reynolds:
C_D of streamlined strut > C_D of cylinder
because most drag is from skin friction



and the streamlined strut has greater surface area

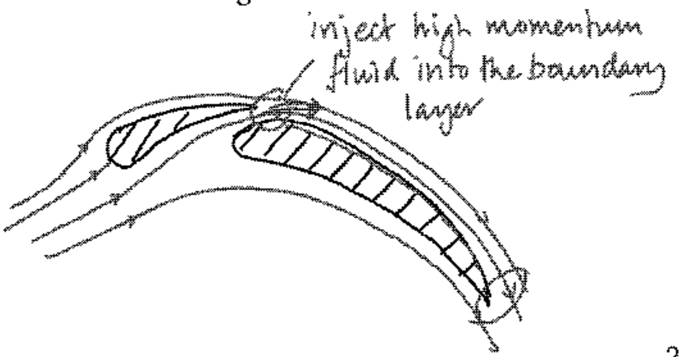
high Reynolds number
C_D of streamlined strut < C_D of cylinder
because most drag is form drag



Streamlining often has the side-effect of increasing the skin friction drag. However, this is only influential at low Reynolds numbers. The drag coefficients for a flat plate, a cylinder and a streamlined strut are shown above. Note that C_D for a cylinder is generally greater than that of a sphere.

8.7 Drag reduction - other methods

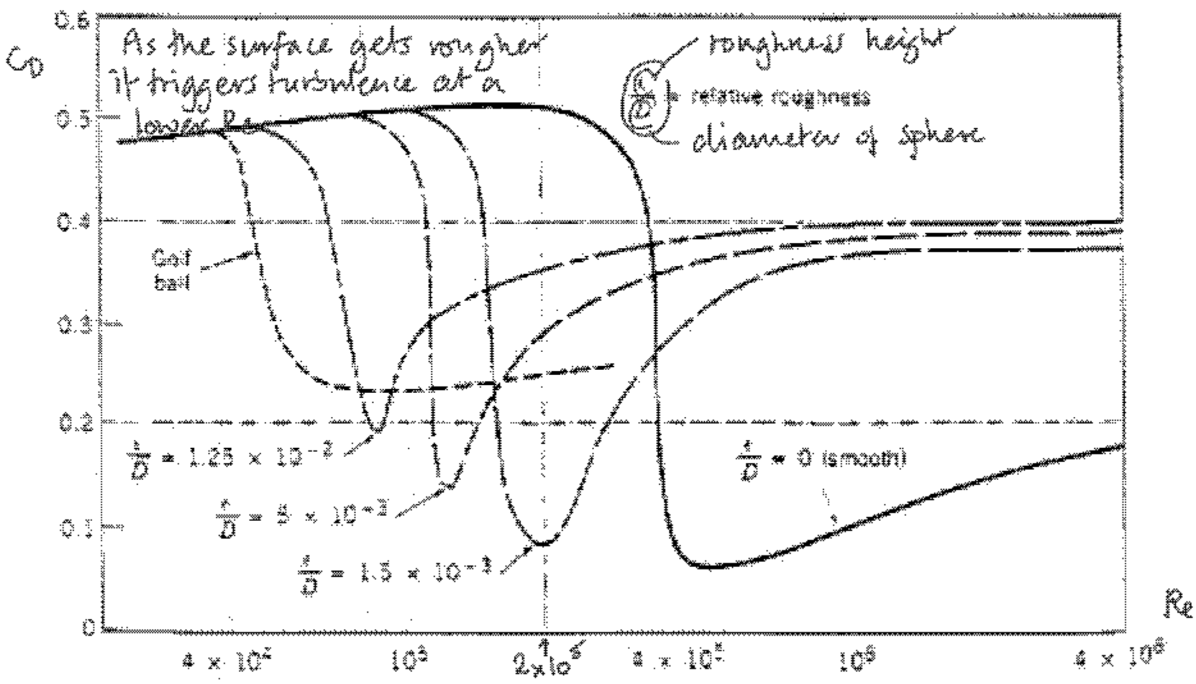
In chapter 4 we saw two ways to delay boundary layer separation and therefore reduce form drag:



1. Inject high momentum air into the boundary layer. This works well and is the idea behind slats at the front of aircraft wings for high angle of attack operation.

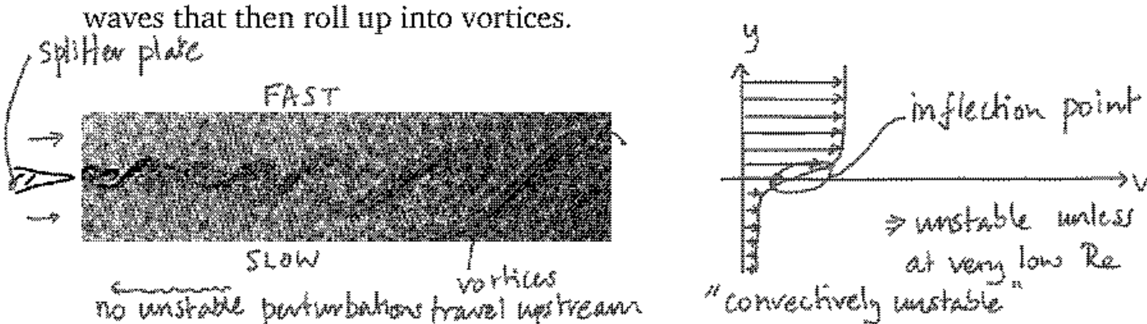
2. Drain away the layer of slow-moving air at the bottom of the boundary layer by sucking it through small holes in the object. This works well but often requires more power than is saved by reducing form drag.

Another way to reduce drag is to trigger turbulence in the boundary layer by roughening the surface of the body.

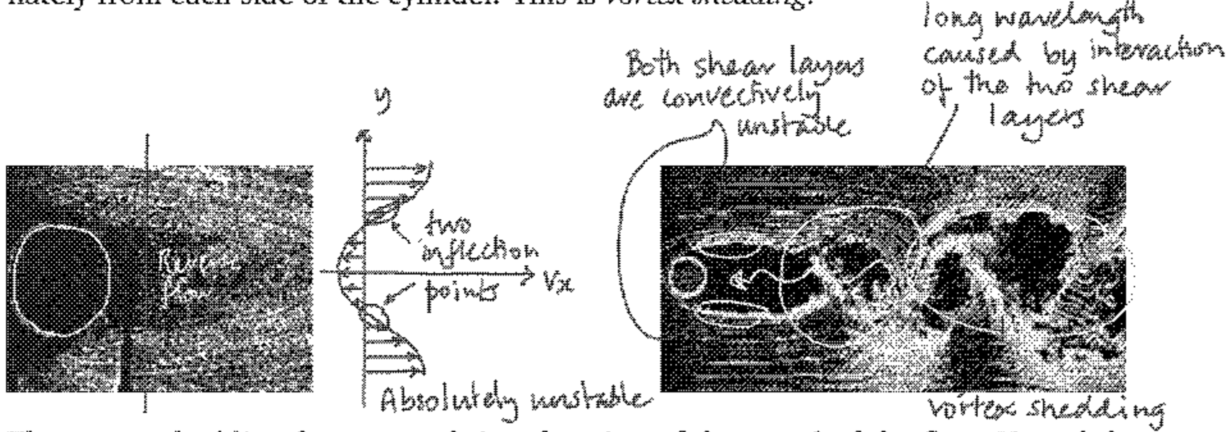


8.8 Flow instability and vortex shedding

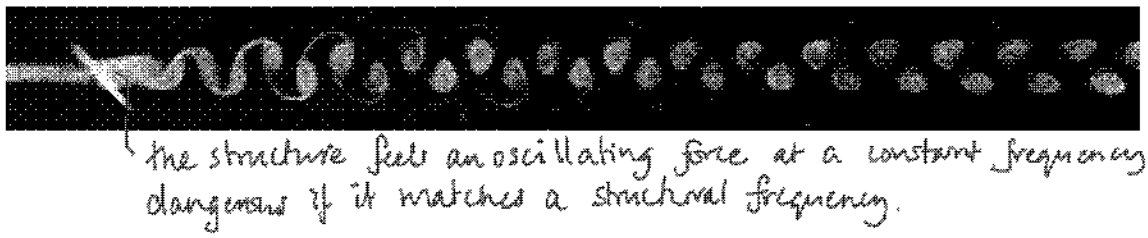
When boundary layers separate they create a *shear layer*. Shear layers are inherently unstable because they have inflexion points in their velocity profiles. They develop waves that then roll up into vortices.



There are two approximately-parallel shear layers behind a bluff body such as a cylinder. On the outside, the flow goes forwards. On the inside, it goes backwards. This situation is even more unstable than a single shear layer. The shear layers start by snaking up and down together. Soon they roll up into vortices that are shed alternately from each side of the cylinder. This is *vortex shedding*.

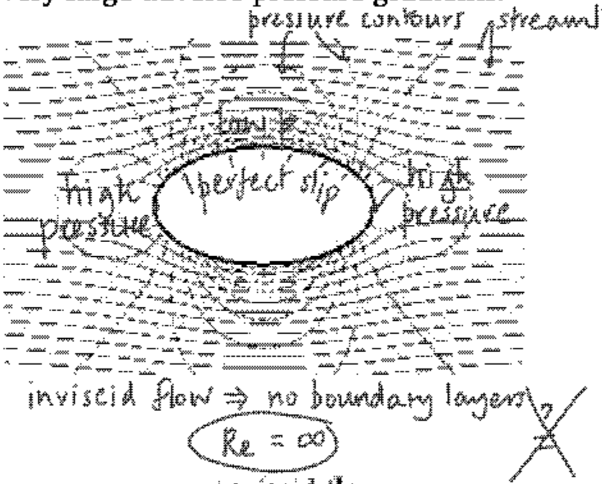


The vortex shedding frequency, f, is a function of the speed of the flow, V, and the distance between the shear layers, D. Experimentally one finds that the *Strouhal number*, fD/V, is approximately 0.2 at moderate and high Reynolds numbers. Vortex shedding has important consequences for structures such as chimneys, particularly if the frequency of vortex shedding matches the resonant frequency of the structure.

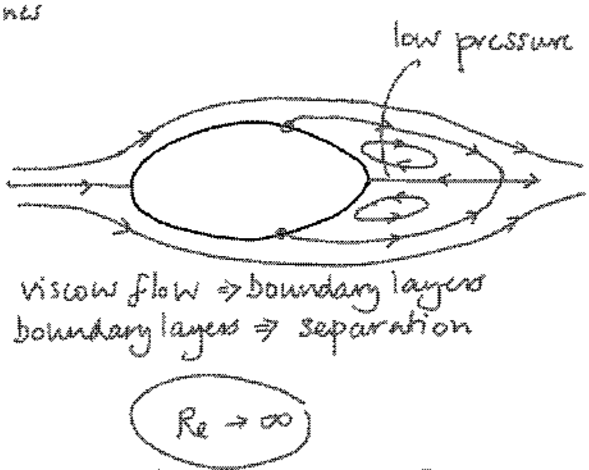


8.9 Relevance of inviscid flow

By definition, inviscid fluids have no viscosity. Consequently there is perfect slip at a solid boundary and there can be no boundary layers. Obviously, if there are no boundary layers, there can be no boundary layer separation, even in the presence of very large adverse pressure gradients.



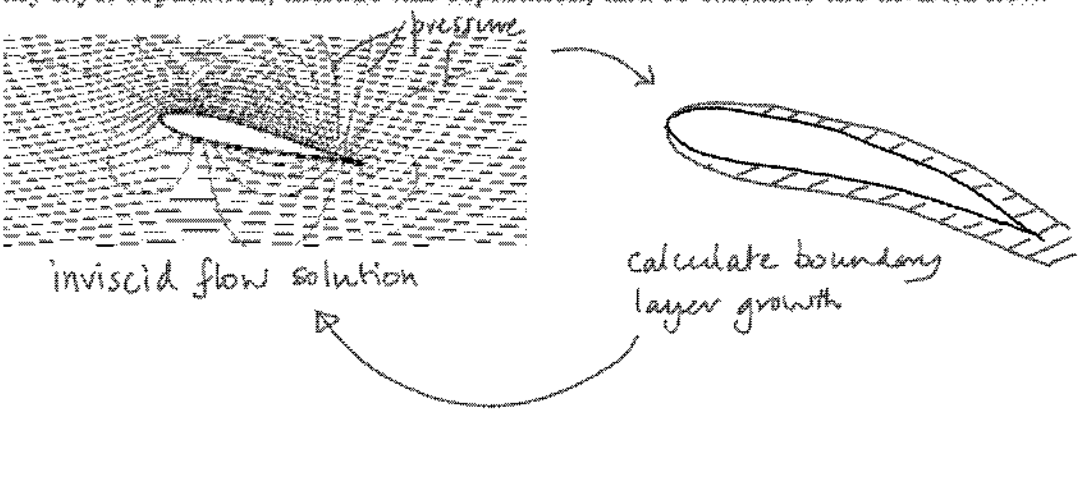
Inviscid flow
no boundary layers



High Reynolds number flow
boundary layers separate

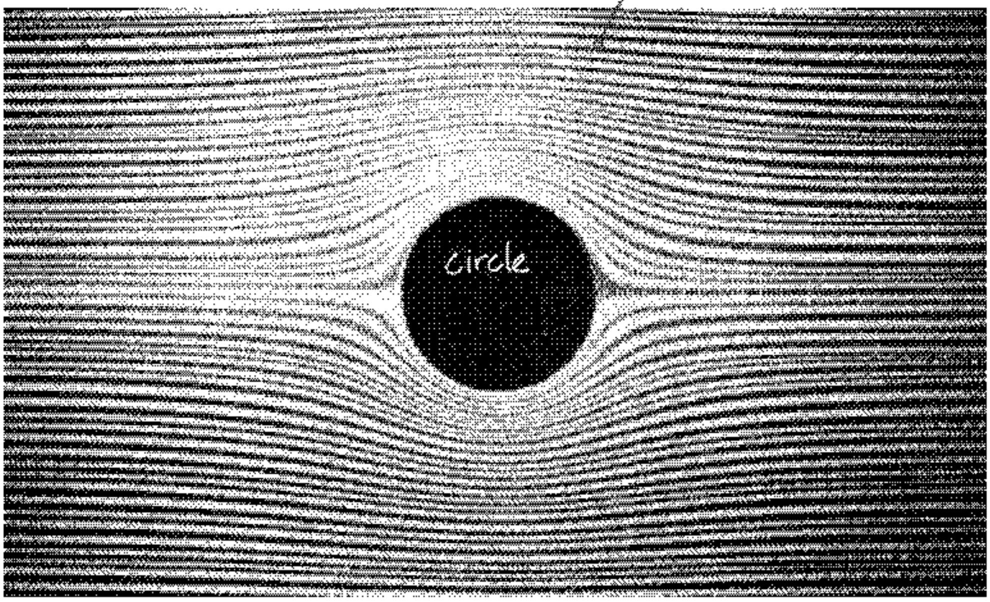
It is tempting to think of inviscid flow as being the solution when the Reynolds number tends to infinity *but this is not the case!* Inviscid flows are only realistic in regions where there is a *favourable* pressure gradient, i.e. where the boundary layers do not separate.

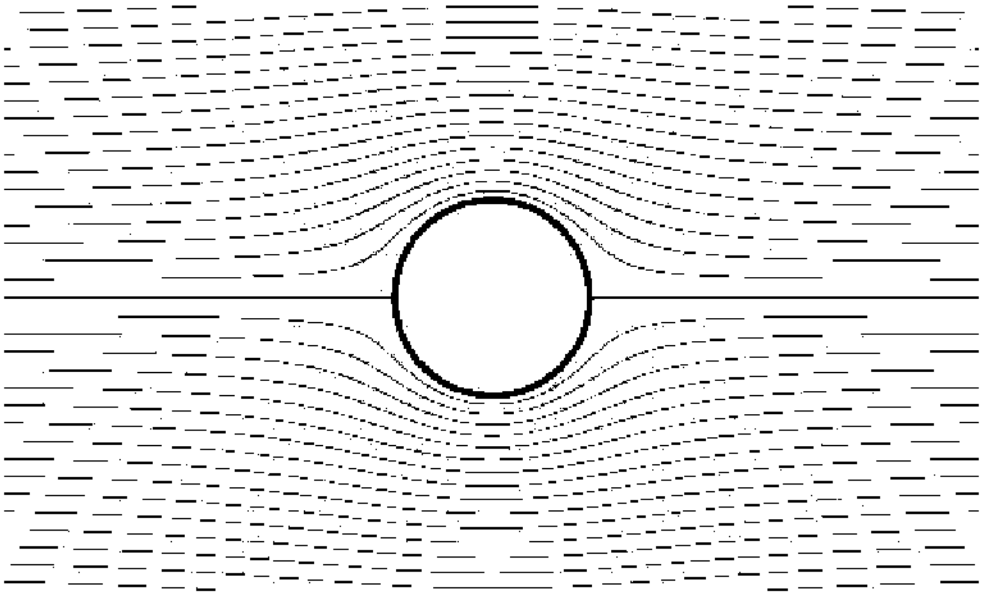
Nevertheless, it is often much easier to calculate the inviscid flow than the viscous flow. We then examine the pressure gradients, work out where there will be boundary layer separation, include this separation, and re-calculate the inviscid flow.



8.10 HELE-SHAW CELLS

In a Hele-Shaw cell, a viscous fluid is forced around objects that are sandwiched between two flat plates. The Reynolds number is very small. However, in this very special configuration, the streamlines just happen to be the same as the streamlines that would be obtained for the same inviscid flow. Hence, Hele-Shaw cells are often used to demonstrate the streamlines of inviscid flows.





This can be shown by comparing the Navier-Stokes equation for the Hele-Shaw cell with the Euler equation for 2D inviscid flow. In both cases the velocity can be expressed as a velocity potential, ϕ. For the same boundary conditions, the two equations have the same flow solutions.